BOARD OF STUDIES
NEW SOUTH WALES

Mathematics General

Stage 6 Syllabus

2012

Published by
Board of Studies NSW
GPO Box 5300
Sydney NSW 2001
Australia

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Contents

* Satisfactory completion of the Preliminary Mathematics General course may be followed by study of either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course. (Note: The HSC Mathematics General 2 course is a Board Developed Course (examined at the HSC), while the HSC Mathematics General 1 course is a Content Endorsed Course (not examined at the HSC)).

1 The Higher School Certificate Program of Study

The purpose of the Higher School Certificate program of study is to:

- provide a curriculum structure that encourages students to complete secondary education
- foster the intellectual, social and moral development of students, in particular developing their:
  - knowledge, skills, understanding and attitudes in the fields of study they choose
  - capacity to manage their own learning
  - desire to continue learning in formal or informal settings after school
  - capacity to work together with others
  - respect for the cultural diversity of Australian society
- provide a flexible structure within which students can prepare for:
  - further education and training
  - employment
  - full and active participation as citizens
- provide formal assessment and certification of students' achievements
- provide a context within which schools also have the opportunity to foster students' physical and spiritual development.

2 Rationale

Mathematics is deeply embedded in modern society. From the numeracy skills required to manage personal finances, to making sense of data in various forms, to leading-edge technologies in the sciences and engineering, mathematics provides the framework for interpreting, analysing and predicting, and the tools for effective participation in an increasingly complex society.

The need to interpret the large volumes of data made available through technology draws on skills in logical thought and skills in checking claims and assumptions in a systematic way. Mathematics is the appropriate training ground for the development of these skills. The thinking required to enhance further the power and usefulness of technology in real-world applications requires advanced mathematical training. The rapid advances in technology experienced in recent years have driven, and been driven by, advances in the discipline of mathematics.

The development of mathematics throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. In this way, the discipline has continued to evolve through a process of observation, conjecture, proof and application.

Effective participation in a changing society is enhanced by the development of mathematical competence in contextualised problem-solving. Experience in such problem-solving is gained by students gathering, analysing and interpreting mathematical information, and applying mathematics to model situations.

The opportunities for creative thinking, communication and contextualised problem-solving within the Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course assist students in finding solutions for the broad range of problems encountered in life beyond secondary schooling.

The purpose of the courses is to provide an appropriate mathematical background for students who wish to enter occupations that require the use of a variety of mathematical and statistical techniques. As well as introducing some new mathematical content, the various Focus Studies within the courses give students the opportunity to apply, and develop further, the knowledge, skills and understanding initially developed in the various Strands: Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling. Through the Focus Studies, students develop the capacity to integrate their knowledge, skills and understanding across the Strands in contemporary contexts chosen for their ongoing relevance to the students' everyday lives and likely vocational pathways.

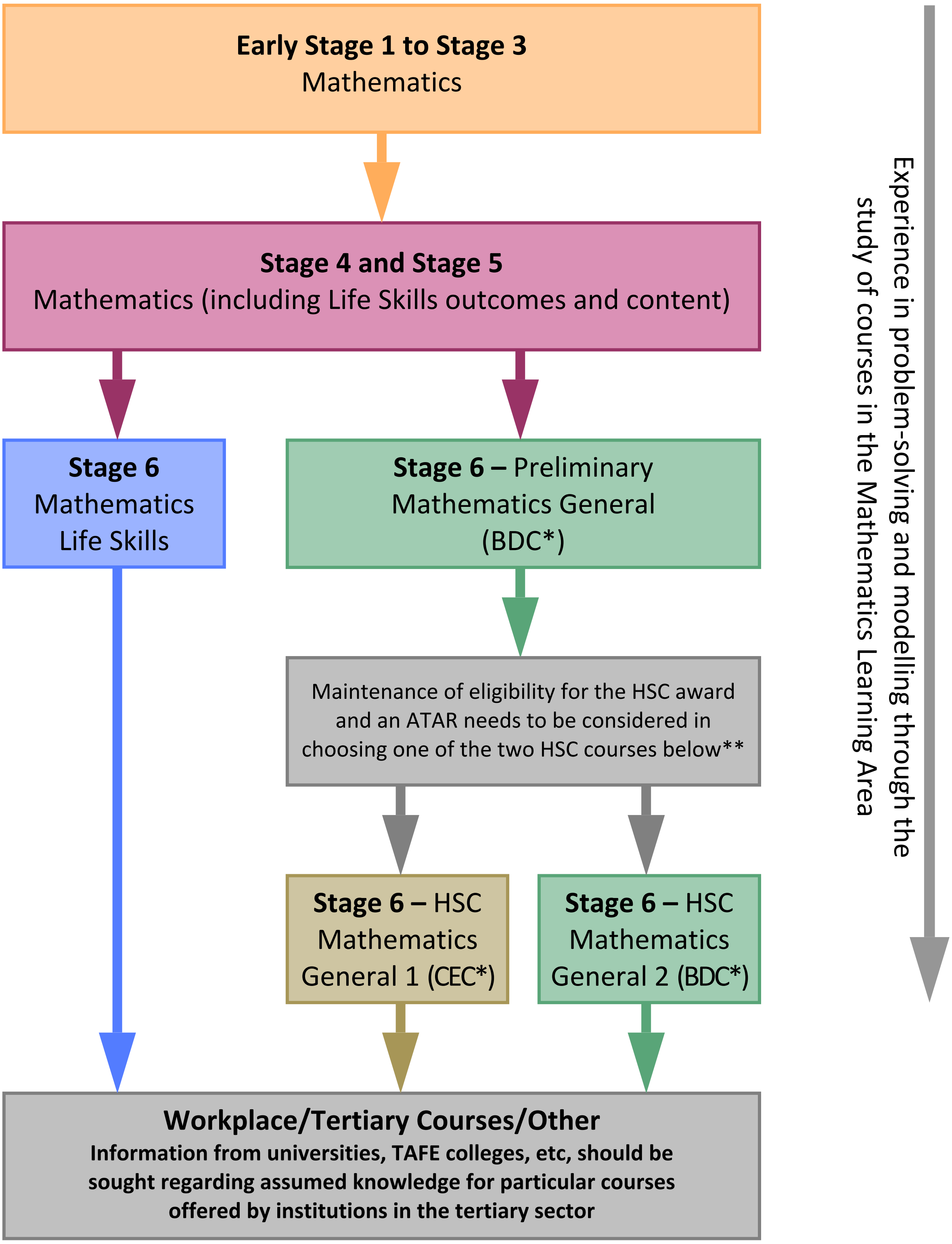
The Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course are highly contextualised. The Preliminary Mathematics General course contains the five Strands as well as two Focus Studies: Mathematics and Communication, and Mathematics and Driving. It is structured to provide appropriate pathways to both the HSC Mathematics General 2 course and the HSC Mathematics General 1 course.

Preliminary Mathematics General provides an appropriate course of study for students who have demonstrated competence in mathematics up to and including at least Stage 5.1 by the end of Year 10. The two Focus Studies within the course have been designed for one-third of the course time.

The HSC Mathematics General 2 course has been written on the assumption that students have demonstrated a high level of competence in the Preliminary Mathematics General course. The two Focus Studies within the course – Mathematics and Health, and Mathematics and Resources – have been designed for one-third of the course time. The course provides students with the opportunity to develop an understanding of and competence in further aspects of mathematics through a large variety of real-world applications for a range of concurrent HSC studies, such as in the life sciences, the humanities and business studies. The course also provides a strong foundation for vocational pathways, in the workforce and in further training, and for university courses in the humanities, nursing and paramedical sciences.

The HSC Mathematics General 1 course has been written to meet the needs of students who have demonstrated competence in the Preliminary Mathematics General course. The four Focus Studies within the course – Mathematics and Design, Mathematics and Household Finance, Mathematics and the Human Body, and Mathematics and Personal Resource Usage – have been designed for two-thirds of the course time. The course provides students with the opportunity to develop an understanding of and competence in further aspects of mathematics through a large variety of real-world applications for concurrent HSC studies, such as in vocational education and training courses, other practically oriented courses, and some humanities courses, and for vocational pathways, in the workforce or in further training.

3 Continuum of Learning for Stage 6 Mathematics General Students



* BDC – Board Developed Course (HSC BDCs are examined at the HSC).
CEC – Content Endorsed Course (HSC CECs are not examined at the HSC).

** The HSC Mathematics General 1 course (two units of study in the HSC Year) is a Content Endorsed Course and cannot be used to meet the requirement that, to be eligible for the HSC award, students must study at least six units of Board Developed Courses. Also, the two units of study for the HSC Mathematics General 1 course cannot be counted in the 10 units required for the calculation of an ATAR. For further information, please refer to the Board's Assessment Certification Examination (ACE) website at http://ace.bos.nsw.edu.au.

4 Mathematics in Stage 6

4.1 Courses

For the Higher School Certificate, there are three Board Developed mathematics courses for study as Preliminary Year courses: (in increasing order of difficulty) Preliminary Mathematics General, Preliminary Mathematics (‘2 Unit’), and Preliminary Mathematics Extension. There are four Board Developed Courses and one Content Endorsed Course (CEC) for study as HSC Year courses: (in increasing order of difficulty) Mathematics General 1 (CEC), Mathematics General 2, Mathematics (‘2 Unit’), Mathematics Extension 1, and Mathematics Extension 2.

Students of the two Mathematics General pathways study the preliminary course, Preliminary Mathematics General, followed by either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.

Mathematics (‘2 Unit’) consists of the courses Preliminary Mathematics (‘2 Unit’) and HSC Mathematics (‘2 Unit’). Students studying one or both Extension courses study the course, Preliminary Mathematics Extension, before undertaking the study of HSC Mathematics Extension 1, or HSC Mathematics Extension 1 and HSC Mathematics Extension 2.

The following assumptions and recommendations regarding Stage 5 Mathematics, typically undertaken by students in Years 9 and 10, are provided in relation to the suite of Stage 6 Mathematics courses.

The Preliminary Mathematics General course has been constructed on the assumption that students have studied the content and achieved the outcomes of the *Mathematics Years 7–10 Syllabus* (2002) up to, and including, the content and outcomes of Stage 5.1. For students who intend to study the HSC Mathematics General 2 course, it is recommended that they experience at least some of the Stage 5.2 content, particularly the *Patterns and Algebra* topics and *Trigonometry*, if not all of the content.

The Mathematics (‘2 Unit’) course has been constructed on the assumption that students have studied the content and achieved the outcomes of the *Mathematics Years 7–10 Syllabus* (2002) up to, and including, the content and outcomes of Stage 5.2. Where possible, it is recommended that they also experience the topics *Real Numbers*, *Algebraic Techniques* and *Coordinate Geometry,* as well as at least some of *Trigonometry* from Stage 5.3 (identified by §), if not all of the content.

The Preliminary Mathematics Extension and HSC Mathematics Extension 1 courses have been constructed on the assumption that students have studied the content and achieved the outcomes of the *Mathematics Years 7–10 Syllabus* (2002) up to, and including, the content and outcomes of Stage 5.3. Where possible, it is recommended that they also experience the optional topics (identified by #) *Curve Sketching and Polynomials*, *Functions and Logarithms*, and *Circle Geometry*.

The Mathematics Extension 2 course consists of an HSC course (only) and may be undertaken following completion of the Preliminary Mathematics (‘2 Unit’) and Preliminary Mathematics Extension courses. Students may study the Mathematics Extension 2 course concurrently with, or following completion of, the HSC Mathematics Extension 1 course.

The preliminary and HSC courses undertaken by students who study the Preliminary Mathematics General/HSC Mathematics General 1 pathway, the Preliminary Mathematics General/HSC Mathematics General 2 pathway, or Mathematics (‘2 Unit’), and by students who study mathematics in Stage 6 to Mathematics Extension 1 or Mathematics Extension 2 level, are illustrated on the following pages.

Preliminary Mathematics General/HSC Mathematics General 1 pathway

Preliminary *Mathematics General** Units: 2 Indicative hours: 120	→	*HSC* *Mathematics General 1* Units: 2 Indicative hours: 120

Total indicative hours: 240

Preliminary Mathematics General/HSC Mathematics General 2 pathway

Preliminary *Mathematics General** Units: 2 Indicative hours: 120	→	*HSC* *Mathematics General 2* Units: 2 Indicative hours: 120

Total indicative hours: 240

Mathematics (‘2 Unit’)

Preliminary *Mathematics (‘2 Unit’)* Units: 2 Indicative hours: 120	→	*HSC* *Mathematics (‘2 Unit’)* Units: 2 Indicative hours: 120

Total indicative hours: 240

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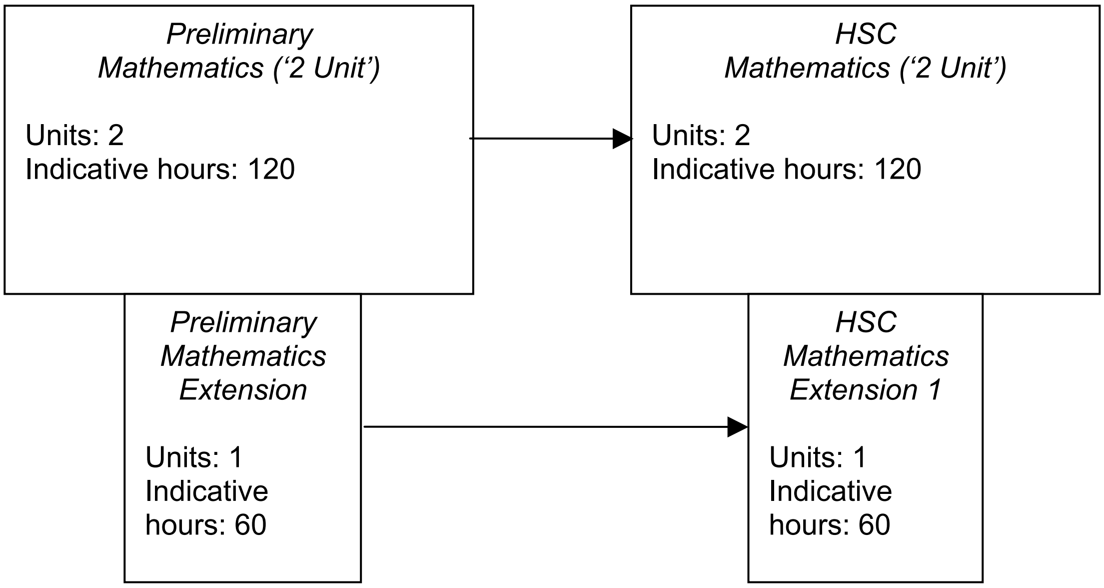
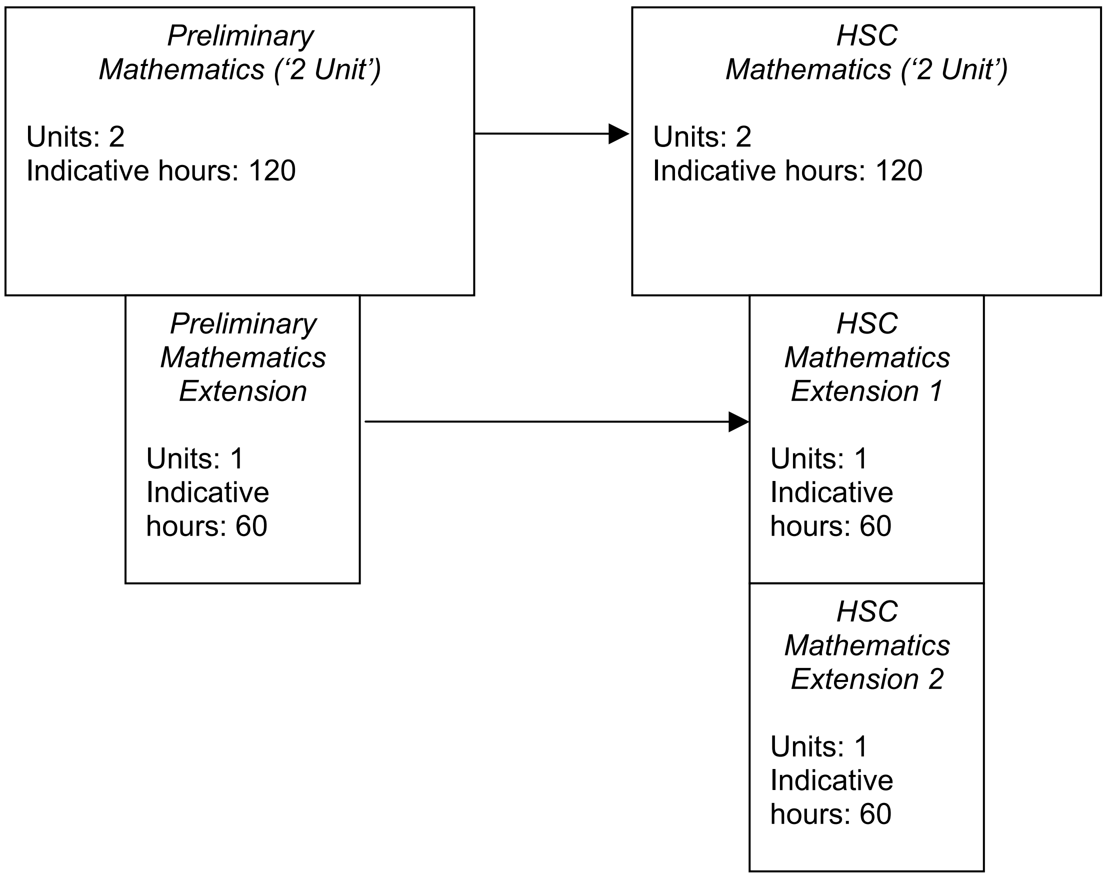
Stage 6 Mathematics to Mathematics Extension 1 level



Total indicative hours: 360

Stage 6 Mathematics to Mathematics Extension 2 level



Total indicative hours: 420

4.2 Statistics in the Mathematics General Syllabus

The study of Statistics in the Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course in Stage 6 recognises the need for people in contemporary society to develop competence in understanding, interpreting and analysing information, or ‘data’, displayed in tabular or graphical forms. Such competence, often described as ‘statistical literacy’, has become more and more important with advances in technology and the resulting increases in the availability of large amounts of data. Students need to recognise that while such data is very often used scrupulously and with high levels of accuracy as a means of persuasion to a particular point of view, it can also be used in a deliberately misleading manner in order to deliver false conclusions.

Statistics in the Mathematics General syllabus builds on the Data strand in the *Mathematics K–6 Syllabus* (2002) and the *Mathematics Years 7–10 Syllabus* (2002). The strand extends from Early Stage 1 to Stage 5.2 and provides students with the opportunity to develop knowledge, skills and understanding in the collection, organisation, display and analysis of data. Students should be aware that mathematics and statistics are different fields of study, but that various aspects of mathematics are fundamental to the design and application of statistical techniques. They should also develop an understanding that mathematics is concerned with describing patterns and relationships and developing generalisations in situations of certainty, whereas Statistics is concerned with describing patterns and relationships in situations of uncertainty, and developing generalisations without an assurance of complete accuracy.

The principal focus of Statistics as a practical discipline is the application of techniques for the detection and interpretation of systematic patterns in data drawn from the real world. The discipline can be divided into Descriptive Statistics and Inferential Statistics. The study of Statistics within the Mathematics General syllabus aims to develop further students’ knowledge, skills and understanding in Descriptive Statistics. Study of Inferential Statistics, which has foundations in Descriptive Statistics and Probability, is beyond the scope of the syllabus.

Mathematical models of real-world situations are a key feature of each of the Stage 6 Mathematics courses. Many of these models are of a ‘deterministic’ nature, ie either they are models that represent situations that are certain, or they are models that represent uncertain situations where the uncertainty is not taken into account. They are called ‘deterministic’ models because the values assigned to a model’s inputs completely determine the model’s output: there is no influence from chance events.

However, there are many real-world situations where the use of such models is inappropriate. Sometimes it is possible to gather data only from a small sample of a larger population in order to make predictions about the behaviour of the population. At other times, the situation being studied involves behaviour that is random in nature, eg the motion of molecules in a gas. In such cases, the use of methods drawn from the study of Statistics is often an appropriate way of extracting usable information from the data gathered.

The study of topics in Descriptive Statistics complements the study of the other components of the Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course, because it provides a foundation for extending the types of practical situations with which these courses are concerned to situations where the impact of uncertainty is recognised.

5 Aim

The Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course are designed to promote the development of knowledge, skills and understanding in areas of mathematics that have direct application to the broad range of human activity, including a range of post-school pathways requiring a variety of mathematical and statistical techniques.

Students will learn to use a range of techniques and tools, including relevant technologies, in order to develop solutions to a wide variety of problems relating to their present and future needs and aspirations.

The HSC Mathematics General 2 course provides a strong foundation for a broad range of vocational pathways, as well as for a range of university courses. The HSC Mathematics General 1 course provides an appropriate foundation for a range of such vocational pathways, either in the workforce or in further training.

6 Objectives

Knowledge, skills and understanding

Students will develop the ability to:

- apply reasoning, and the use of appropriate language, in the evaluation and construction of arguments and the interpretation and use of models based on mathematical and statistical concepts
- use concepts and apply techniques to the solution of problems in algebra and modelling, measurement, financial mathematics, data and statistics, and probability
- use mathematical skills and techniques, aided by appropriate technology, to organise information and interpret practical situations
- interpret and communicate mathematics in a variety of written and verbal forms, including diagrams and statistical graphs.

Values and attitudes

Students will develop:

- appreciation of the relevance of mathematics.

7 Objectives and Outcomes

Knowledge, skills and understanding

Objectives	Preliminary Mathematics General Outcomes	HSC Mathematics General 2 Outcomes	HSC Mathematics General 1 Outcomes
Students will develop the ability to:	**A student:**	**A student:**	**A student:**
apply reasoning, and the use of appropriate language, in the evaluation and construction of arguments and the interpretation and use of models based on mathematical and statistical concepts	MGP-1 uses mathematics and statistics to compare alternative solutions to contextual problems	MG2H-1 uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts	MG1H-1 uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
	MGP-2 represents information in symbolic, graphical and tabular form	MG2H-2 analyses representations of data in order to make inferences, predictions and conclusions	MG1H-2 analyses representations of data in order to make predictions
use concepts and apply techniques to the solution of problems in algebra and modelling, measurement, financial mathematics, data and statistics, and probability	MGP-3 represents the relationships between changing quantities in algebraic and graphical form	MG2H-3 makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions	MG1H-3 makes predictions about everyday situations based on simple mathematical models
	MGP-4 performs calculations in relation to two-dimensional and three-dimensional figures	MG2H-4 analyses two-dimensional and three-dimensional models to solve practical problems, including those involving spheres and non-right-angled triangles	MG1H-4 analyses simple two-dimensional and three-dimensional models to solve practical problems
	MGP-5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units	MG2H-5 interprets the results of measurements and calculations and makes judgements about reasonableness, including the degree of accuracy of measurements and calculations and the conversion to appropriate units	MG1H-5 interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units
	MGP-6 models financial situations relevant to the student's current life using appropriate tools	MG2H-6 makes informed decisions about financial situations, including annuities and loan repayments	MG1H-6 makes informed decisions about financial situations likely to be encountered post-school
	MGP-7 determines an appropriate form of organisation and representation of collected data	MG2H-7 answers questions requiring statistical processes, including the use of the normal distribution, and the correlation of bivariate data	MG1H-7 develops and carries out simple statistical processes to answer questions posed
	MGP-8 performs simple calculations in relation to the likelihood of familiar events	MG2H-8 solves problems involving counting techniques, multistage events and expectation	MG1H-8 solves problems involving uncertainty using basic counting techniques

Objectives	Preliminary Mathematics General Outcomes	HSC Mathematics General 2 Outcomes	HSC Mathematics General 1 Outcomes
Students will develop the ability to:	**A student:**	**A student:**	**A student:**
use mathematical skills and techniques, aided by appropriate technology, to organise information and interpret practical situations	MGP-9 uses appropriate technology to organise information from a limited range of practical and everyday contexts	MG2H-9 chooses and uses appropriate technology to locate and organise information from a range of contexts	MG1H-9 chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
interpret and communicate mathematics in a variety of written and verbal forms, including diagrams and statistical graphs	MGP-10 justifies a response to a given problem using appropriate mathematical terminology	MG2H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response	MG1H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others

Values and attitudes

Objectives	Preliminary Mathematics General Outcomes	HSC Mathematics General 2 Outcomes	HSC Mathematics General 1 Outcomes
Students will develop:	**A student:**	**A student:**	**A student:**
appreciation of the relevance of mathematics	MGP-VA develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation	MG2H-VA appreciates the importance of mathematics in everyday life and its usefulness in contributing to society	MG1H-VA appreciates the importance of mathematics in everyday life and its usefulness in contributing to society

8 Preliminary Mathematics General/ HSC Mathematics General 2 Pathway

8.1 Pathway Structure

The following overview illustrates the structure of the Preliminary Mathematics General/HSC Mathematics General 2 pathway.

Preliminary Mathematics General Course*		**HSC Mathematics General 2 Course**	
Strand:	**Financial Mathematics**	**Strand:**	**Financial Mathematics**
FM1	Earning and managing money	FM4	Credit and borrowing
FM2	Investing money	FM5	Annuities and loan repayments
FM3	Taxation		
Strand:	**Data and Statistics**	**Strand:**	**Data and Statistics**
DS1	Statistics and society, data collection and sampling	DS4	Interpreting sets of data
DS2	Displaying and interpreting single data sets	DS5	The normal distribution
DS3	Summary statistics	DS6	Sampling and populations
Strand:	**Measurement**	**Strand:**	**Measurement**
MM1	Units of measurement and applications	MM4	Further applications of area and volume
MM2	Applications of perimeter, area and volume	MM5	Applications of trigonometry
MM3	Similarity of two-dimensional figures, right-angled triangles	MM6	Spherical geometry
Strand:	**Probability**	**Strand:**	**Probability**
PB1	Relative frequency and probability	PB2	Multistage events and applications of probability
Strand:	**Algebra and Modelling**	**Strand:**	**Algebra and Modelling**
AM1	Algebraic manipulation	AM3	Further algebraic skills and techniques
AM2	Interpreting linear relationships	AM4	Modelling linear relationships
		AM5	Modelling non-linear relationships
Focus Study:	**Mathematics and Communication**	**Focus Study:**	**Mathematics and Health**
FSCo1	Mobile phone plans	FSHe1	Body measurements
FSCo2	Digital download and file storage	FSHe2	Medication
		FSHe3	Life expectancy
Focus Study:	**Mathematics and Driving**	**Focus Study:**	**Mathematics and Resources**
FSDr1	Costs of purchase and insurance	FSRe1	Water availability and usage
FSDr2	Running costs and depreciation	FSRe2	Dams, land and catchment areas
FSDr3	Safety	FSRe3	Energy and sustainability

* The Preliminary Mathematics General course is undertaken by all students intending to study either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.

8.2 Employability Skills

The Employability Skills build on the Mayer Key Competencies (developed in 1992), which attempted to describe generic competencies for effective participation in work. The *Employability Skills for the Future* report (March 2002) indicated that business and industry required a broader range of skills than the Mayer Key Competencies Framework. It featured an Employability Skills Framework identifying eight Employability Skills: *communication, teamwork, problem-solving, initiative and enterprise, planning and organising, self-management, learning,* and *technology.*

The Preliminary Mathematics General/HSC Mathematics General 2 pathway provides a context within which to develop general competencies considered essential for the acquisition of effective, higher-order thinking skills necessary for further education, work and everyday life.

Employability skills are embedded in the Preliminary Mathematics General and HSC Mathematics General 2 courses to enhance student learning. The employability skills are developed through the methodologies of the syllabus and through classroom pedagogy and reflect core processes of mathematical inquiry undertaken by students.

At all levels of the courses, students are developing the employability skill *learning*. As they engage with the various topics in the courses and related applications and modelling tasks, the employability skills *planning and organisation, communication, self-management, teamwork* and *initiative and enterprise* are developed. Through relevant course content and the advice provided on the selection and use of appropriate technology, students can develop the employability skill *technology*. Finally, students' continual involvement with seeking solutions to problems, both large and small, contributes towards their development of the employability skill *problem-solving*.

8.3 Use of Technology

(a) In learning and teaching, and school-based assessment

The appropriateness, viability and level of use of different types of technology in the learning and teaching of courses within the Mathematics Key Learning Area are decisions for students, teachers and schools. However, the use of technology is encouraged in the learning and teaching, and school-based assessment, where appropriate, of courses within the learning area.

The Preliminary Mathematics General and HSC Mathematics General 2 courses contain advice and suggestions in relation to the use of a range of technology in the 'Use of technology', 'Considerations' and 'Suggested applications' sections within the course content. The courses provide a range of opportunities for the use of calculators and computer software packages in learning and teaching. This includes opportunities to utilise the graphing functions and financial and statistical capabilities of calculators, spreadsheets, and dynamic geometry and statistics software packages.

(b) In the HSC examinations

In the HSC examinations for the Stage 6 HSC Mathematics General 2, HSC Mathematics ('2 Unit'), HSC Mathematics Extension 1 and HSC Mathematics Extension 2 courses, candidates will be permitted to use only calculators manufactured to meet a clear set of requirements regarding functions and capabilities. These functions and capabilities are consistent with and support the knowledge, skills and understanding that students should be able to demonstrate after completing a course, or courses.

8.4 Course Requirements

The Preliminary Mathematics General/HSC Mathematics General 2 pathway consists of a preliminary course, Preliminary Mathematics General, of 120 indicative hours and an HSC course, HSC Mathematics General 2, of 120 indicative hours.

The Preliminary Mathematics General course is constructed on the assumption that students have experienced all of the Stage 5.1 content of the *Mathematics Years 7–10 Syllabus* (2002). Completion of the Preliminary Mathematics General course is a prerequisite for the study of the HSC Mathematics General 2 course (and the HSC Mathematics General 1 course).

Students may not study the Preliminary Mathematics General course or the HSC Mathematics General 2 course (or the HSC Mathematics General 1 course) in conjunction with any other mathematics course in Stage 6.

8.5 Presentation of Content

The course content for the Preliminary Mathematics General/HSC Mathematics General 2 pathway (and for the Preliminary Mathematics General/HSC Mathematics General 1 pathway) is organised into Strands and Focus Studies, with each of the Strands – Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling – divided into *topics* that lead into the Focus Studies.

The Focus Studies are designed to be programmed over a continuous time period, as they provide students with the opportunity to apply, and develop further, the knowledge, skills and understanding initially developed in the Strands, as well as introducing some new mathematical content. It is intended that students develop, through the Focus Studies, the capacity to integrate their knowledge, skills and understanding across the Strands.

The Preliminary Mathematics General course is undertaken by all students intending to study either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course. The Preliminary Mathematics General course includes two Focus Studies: Mathematics and Communication, and Mathematics and Driving.

There are two Focus Studies in the HSC Mathematics General 2 course: Mathematics and Health, and Mathematics and Resources. (There are four Focus Studies in the HSC Mathematics General 1 course: Mathematics and Design, Mathematics and Household Finance, Mathematics and the Human Body, and Mathematics and Personal Resource Usage.)

The course content for the Preliminary Mathematics General/HSC Mathematics General 2 pathway (and for the Preliminary Mathematics General/HSC Mathematics General 1 pathway) is presented in the following format:

1. Initial facing pages for a Strand or Focus Study

Left page

Name of Strand or Focus Study

A summary of the purpose/content of the Strand or Focus Study

Outcomes addressed

A list of course outcomes addressed in the study of the topic(s) in the Strand or Focus Study

Content summary

A list of the topic(s) studied within the Strand or Focus Study

Right page

Terminology

A list of key words and/or phrases met in the Strand or Focus Study, some of which may be new to students

Use of technology

Advice about the nature and use of technology that is appropriate to the teaching and learning of the topic(s) in the Strand or Focus Study

Notes

Notes relevant to teaching particular aspects of the Strand or Focus Study

2. Subsequent facing pages for a Strand or Focus Study

Left page	***Right page***
Name of Topic A summary of the purpose/content of the topic ***Outcomes addressed*** A list of course outcomes addressed in the study of the topic ***Content*** *Students:* The mathematical content to be addressed in the topic	***Considerations*** A list of important considerations for teaching and learning the topic ***Suggested applications*** A list of examples indicating the types of applications used to introduce and illustrate the mathematical content of the topic

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9 Preliminary Mathematics General Course Content*

Hours shown are indicative only.

Strands 80 hours

Financial Mathematics

FM1 Earning and managing money
FM2 Investing money
FM3 Taxation

Data and Statistics

DS1 Statistics and society, data collection and sampling
DS2 Displaying and interpreting single data sets
DS3 Summary statistics

Measurement

MM1 Units of measurement and applications
MM2 Applications of perimeter, area and volume
MM3 Similarity of two-dimensional figures, right-angled triangles

Probability

PB1 Relative frequency and probability

Algebra and Modelling

AM1 Algebraic manipulation
AM2 Interpreting linear relationships

Focus Studies 40 hours

Mathematics and Communication

FSCo1 Mobile phone plans
FSCo2 Digital download and file storage

Mathematics and Driving

FSDr1 Costs of purchase and insurance
FSDr2 Running costs and depreciation
FSDr3 Safety

Total indicative hours 120 hours

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Strand: Financial Mathematics

Financial Mathematics is concerned with earning, using and managing money. The study of Financial Mathematics is important in developing students' ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources effectively.

In the Financial Mathematics Strand in the Preliminary Mathematics General course, students investigate ways in which individuals earn, manage and invest money, and perform a range of calculations, including calculations in relation to taxation and the value of investments.

Outcomes addressed

A student:

MGP-1 uses mathematics and statistics to compare alternative solutions to contextual problems

MGP-2 represents information in symbolic, graphical and tabular form

MGP-3 represents the relationships between changing quantities in algebraic and graphical form

MGP-6 models financial situations relevant to the student's current life using appropriate tools

MGP-9 uses appropriate technology to organise information from a limited range of practical and everyday contexts

MGP-10 justifies a response to a given problem using appropriate mathematical terminology.

Content summary

FM1 Earning and managing money

FM2 Investing money

FM3 Taxation

Terminology

allowance	goods and services tax (GST)	percentage interest rate
annual leave	gross pay/gross income	piecework
annual leave loading	income	principal
appreciated value	income tax	refund
budget	inflation	royalty
commission	initial amount	salary
compound interest	interest	shares
compounding period	interest rate	simple interest
deduction	Medicare levy	sliding scale
dividend	net pay	tax deduction
dividend yield	overtime	tax payable
double time	Pay As You Go (PAYG)	tax scale
earnings	Payment Summary	taxable income
final amount	penalty rate	time-and-a-half
franked dividend	per annum (pa)	wage

Use of technology

Spreadsheets are widely used in the workplace, especially in the business and finance sectors. Students should construct and use spreadsheets to perform particular tasks, including calculation of wages and overtime, preparation of personal and family budgets, and tracking of a share portfolio.

Spreadsheets allow the user to perform 'what if' analyses, eg to compare the interest earned on fixed-term deposits for various periods and interest rates.

Students can use prepared spreadsheets to determine Pay As You Go (PAYG) income tax and the Medicare levy for different taxable incomes.

The internet should be used as a source of up-to-date information, eg interest rates, share prices, and income tax scales.

Notes

In Stage 5, students solved problems involving earning and spending money, and the management of money. In the Financial Mathematics Strand in the Preliminary Mathematics General course, students will develop these concepts further in relation to earning and managing money, investing money, and taxation.

Learning and teaching in the topic areas should be supported by the use of real and up-to-date data, eg calculations of wages and salaries should include calculations in relation to current awards and work contracts.

Students should be familiar with correct terminology in the topic areas and be able to express themselves using this terminology when justifying or explaining their solutions to problems.

FM1 Earning and managing money

The principal focus of this topic is on the calculation of different forms of income and the preparation of personal budgets.

Outcomes addressed

MGP-1, MGP-6, MGP-9, MGP-10

Content

Students:

- calculate monthly, fortnightly, weekly, daily and hourly pay rates from a given salary
- calculate wages involving hourly rates and penalty rates, eg overtime; and special allowances, including allowances for wet work, confined spaces, toxic substances, heat, heights, etc
- describe the differences between salaries, wages and commissions
- compare different ways of earning
- calculate annual leave loading
- calculate earnings based on commission (including commission based on a sliding scale), piecework and royalties
- calculate payments based on government allowances and pensions, eg allowances for youth, tertiary study and travel
- determine deductions from income, eg tax instalments, superannuation contributions, health-fund instalments, union fees and HECS repayments
- calculate net pay following deductions from gross pay
- evaluate a prepared budget
- prepare a budget for a given income, taking into account fixed and discretionary spending.

Considerations

- Calculations involving Youth Allowance, or its equivalent, should be based on the current published rates and conditions. Similarly, the current advertised rates should be used in interest-rate calculations.
- In the Preliminary Mathematics General course, budgeting should be set in personal contexts, as opposed to other contexts such as a family budget. Many students will be familiar with aspects of earning money through employment on a casual or part-time basis. Students should develop weekly and monthly budgets that include income, expenses and savings.

Suggested applications

- Calculate earnings based on prepared timesheets.
- Judy, a cosmetics salesperson, earns a monthly commission of 5% on the first $\$1000$ of sales, 4% on the next $\$2000$ and 3.5% thereafter (ie her commission is based on a sliding scale). Calculate Judy's pay for a month in which her total sales were $\$4800$.
- Make a comparison of wages in various countries for different careers.
- Prepare a personal budget for given income and expenditure. The budget should include a savings plan and should be modified as income and expenses change, eg a change in income or an increase in the cost of petrol. This could be facilitated using a spreadsheet, where it is easy to change the amounts for income and expenses. Students should assess how this affects budgeted savings. They should also explore scenarios where expenses exceed income and how this might be resolved by reducing discretionary spending.
- Review a previously prepared budget to reallocate funds for a sudden contingency.
- Select a career and prepare a report that includes the educational requirements, job conditions, and remuneration. Students can obtain information from sources including newspapers, online job advertisements, and recruitment agencies. The report might also include calculation of net pay after tax.

FM2 Investing money

The principal focus of this topic is the use of formulae and tables to perform calculations related to the value of investments over a period of time.

Outcomes addressed

MGP-1, MGP-2, MGP-3, MGP-6, MGP-9, MGP-10

Content

Students:

- compare costs associated with maintaining accounts with financial institutions
- calculate simple interest
- use tables of values for fixed values of P, and hence draw and describe graphs of I against n for different values of r
- calculate monthly, quarterly and six-monthly interest rates based on quoted rates per annum (pa)
- calculate the final amount, interest and principal using the compound interest formula $A = P(1+r)^n$

  where A (amount) represents the final amount
  P (principal) represents the initial amount
  n is the number of compounding periods
  r is the interest rate per compounding period
- calculate and compare the final amount, interest and principal using a table of compounded values of one dollar (see next page)
- calculate the price of goods following inflation
- calculate the appreciated value of items, eg stamp collections and other memorabilia
- record and graph the price of a share over time
- calculate the dividend paid on a shareholding and the dividend yield (excluding franked dividends)
- compare different investment strategies
- investigate the effect of inflation on prices.

Considerations

- Calculations of simple interest and compound interest should include interest rates expressed other than as 'per annum'.
- Tables of compounded values of $1, such as the table below, are used to calculate the amount to which money invested in a compound interest account has grown in a given period.

	Compounded values of $1				
	Interest rate per period				
Periods	1%	5%	10%	15%	20%
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.461	1.750	2.074

- *For students intending to proceed to the HSC Mathematics General 2 course, teachers should introduce the following terms at this stage: future value (FV) and present value (PV) in the context of the compound interest formula.*

  Note: The 'compounded' value of a dollar is also known as the 'future value' of a dollar. In the financial world, the compound interest formula $A = P(1+r)^n$ is known as the future value formula and is expressed as $FV = PV\left(1+r\right)^n$.

Suggested applications

- An amount of $3000 is invested and compounded annually at 5%. Use the table of compounded values of $1 above to find the value of the investment after three years.

  Solution: Value of investment after three years = 3000×1.158 = $3474
- If the interest rate is quoted as 6% pa, what amount needs to be invested in order for the investment to be worth $850 at year's end?
- Jan and Bob wish to save $10 000 for their granddaughter's university expenses, and to have this amount available in eight years' time. Calculate the single sum they need to invest at 5% pa compounded annually.
- Determine the single sum to be deposited if $10 000 is required in five years' time and a rate of 3% pa, compounded quarterly, is available.
- A principal of $1000 is to be invested for three years. Determine which of the following is the best investment option:

  (i) 6% pa simple interest, (ii) 5.9% pa compounded annually, or (iii) 5.85% pa compounded half-yearly.
- It is predicted that a particular painting will appreciate at a rate of 5% per annum. Calculate its predicted value in 2020 if it was purchased in 2010 for $48 000.
- An investor purchased 1000 shares in a company at $3.98 per share, with a dividend yield of 5.5%. Brokerage costs were 1% of the purchase price. One year later, the shares were sold for $4.80 per share. Calculate the profit after costs. Calculate the loss if the shares had been sold for $2.98 per share.

FM3 Taxation

The principal focus of this topic is on the calculation of tax payable on income and on goods and services.

Outcomes addressed

MGP-2, MGP-3, MGP-6, MGP-9

Content

Students:

- calculate the amount of allowable deductions from gross income
- calculate taxable income
- calculate the Medicare levy (basic levy only – see Tax Pack for details)
- calculate the amount of Pay As You Go (PAYG) tax payable or refund owing, using current tax scales
- calculate the goods and services tax (GST) payable on a range of goods and services
- create and interpret graphs to illustrate and describe different tax rates.

Considerations

- Tax and levy rates, as published on the Australian Taxation Office (ATO) website, should be used in the calculation of income tax and the Medicare levy.
- Goods and services tax (GST) calculations should include finding the original cost of goods before GST was added.
- Allowable tax deductions can include work-related expenses, travel expenses and donations to registered charities. Students should refer to current taxation information and consider tax deductions for different occupations.
- It should be noted that a graph of tax paid against taxable income is a piecewise linear function (see AM2 *Interpreting linear relationships*).

Suggested applications

- Calculate the tax refund (or amount payable) based on a sample Payment Summary, taking into account gross income, tax deductions, taxable income, tax payable on taxable income, the Medicare levy, and tax already paid as per the Payment Summary.
- Students complete a tax return form (as included in the Tax Pack) using a typical PAYG employee's earnings and deductions. The aim is to calculate the 'refund from' or 'amount owed to' the ATO.
- Students use an online tax calculator.

Strand: Data and Statistics

Statistics relates to the collection, display, analysis and interpretation of information, or 'data'. The study of Data and Statistics is important in developing students' appreciation of the contribution that statistical thinking makes to decision-making in society, and in the professional and personal lives of individuals. Students need to develop their ability to critically evaluate data, data displays and statistical results. They need to learn how quantitative data is generated, summarised, presented, modelled and interpreted so that useful conclusions can be drawn.

In the Data and Statistics Strand in the Preliminary Mathematics General course, students focus on the importance of statistical processes and inquiry in society, the planning and management of data collection, the preparation of a variety of data displays, and the calculation of summary statistics for single data sets and their use in interpretation.

Outcomes addressed

A student:

MGP-1	uses mathematics and statistics to compare alternative solutions to contextual problems
MGP-2	represents information in symbolic, graphical and tabular form
MGP-5	demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
MGP-7	determines an appropriate form of organisation and representation of collected data
MGP-9	uses appropriate technology to organise information from a limited range of practical and everyday contexts
MGP-10	justifies a response to a given problem using appropriate mathematical terminology.

Content summary

DS1	Statistics and society, data collection and sampling
DS2	Displaying and interpreting single data sets
DS3	Summary statistics

Terminology

bar chart	histogram	radar chart
bias	interquartile range	random sample
box-and-whisker plot	mean	range
categorical data	measure of location	sample
census	measure of spread	sample size
class interval	median	sector graph (pie chart)
continuous	modal class	spread
cumulative frequency histogram	mode	standard deviation
cumulative frequency polygon	nominal	statistical display
data	ordinal	statistical inquiry
decile	outlying value ('outlier')	stem-and-leaf plot
discrete	percentile	stratified sample
divided bar graph	poll	summary statistics
dot plot	population	systematic sample
five-number summary	quantitative data	ungrouped data
frequency table	quartile	upper extreme/lower extreme
grouped data	questionnaire	upper quartile/lower quartile

Use of technology

Statistical software and spreadsheets should be used in the learning and teaching of the topic areas. Spreadsheets are widely used in the workplace and are a suitable tool for tabulating and graphing data and for calculating summary statistics.

Technology should be used to create frequency tables and statistical graphs, including box-and-whisker plots.

Large and small data sets derived from real-life situations should be accessed. These data sets may be obtained online.

Notes

Learning and teaching, and assessment, materials should use current information from a range of sources, including, but not limited to, surveys, newspapers, journals, magazines, bills and receipts, and the internet.

In their study of statistics and society, students should develop an understanding of the importance of analysing data in planning and decision-making by governments and businesses.

Students may have greater interest in data that relates to their life experiences or to data that they have generated themselves. This data may be collected by survey, measurement or simple experiment. A group surveyed by students may represent the entire population of interest or may represent a sample of the population.

The Australian Bureau of Statistics publishes notes about graph types. Teachers may find these notes useful when giving students experience in the presentation of data displays.

Students could collect, display and analyse data related to a course of study in another key learning area, eg fitness data in PDHPE or attitude data in Geography, or results from a scientific experiment.

DS1 Statistics and society, data collection and sampling

The principal focus of this topic is the planning and management of data collection. Data for this topic may be obtained from published sources or by conducting a survey. The topic includes the use of sampling techniques in order to make inferences about a population. Although the emphasis is on quantitative data, students should be aware of processes related to categorical data.

Outcomes addressed

MGP-1, MGP-2, MGP-7, MGP-10

Content

Students:

- investigate the process of statistical inquiry, and describe the following steps: posing questions, collecting data, organising data, summarising and displaying data, analysing data and drawing conclusions, and writing a report
- identify the target population to be investigated
- determine whether data for the whole population is available (eg the results of a round of a sporting competition), or if sampling is necessary
- classify data as quantitative (either discrete or continuous) or categorical (either nominal or ordinal)
- describe a random sample as a sample in which every member of the population has an equal chance of being included in the sample
- distinguish between the following sample types: random, stratified and systematic, and determine the appropriateness of each type for a given situation
- describe a method for choosing each type of sample in a given situation
- relate sample selection to population characteristics
- identify possible sources of bias in the collection of a sample.

Considerations

- Discussions with students should include:
  - the role of statistical methods in quality control, eg in manufacturing, agriculture, the pharmaceutical industry and medicine
  - issues of privacy and ethics in data collection and analysis
  - the role of organisations that collect and/or use statistics, including the Australian Bureau of Statistics (ABS), the United Nations (UN), and the World Health Organization (WHO).
- Effective questionnaire design includes considerations such as:
  - simple language
  - unambiguous questions
  - requirements for privacy
  - freedom from bias
  - the number of choices of answers for questions, eg if an even number of choices is given, this may force an opinion from the respondents in relation to a particular question, while for other questions it may be appropriate to allow a neutral choice.
- Students should be given a range of opportunities to determine when it is appropriate to use a sample rather than a census, and to determine the best method of sampling in a range of situations.
- Students should recognise that the purpose of a sample is to provide an estimate for a particular population characteristic when the entire population cannot be accessed.
- The generation of random numbers is fundamental to random sampling.
- Examples of classification of data could include gender (male, female) – *categorical, nominal*; quality (poor, average, good, excellent) – *categorical, ordinal*; height (measured in centimetres) – *quantitative, continuous*; school population (measured by counting individuals) – *quantitative, discrete.*
- Consideration needs to be given to the selection of samples, eg for the selection of a stratified sample by age: if 18% of a population is aged under 20, a selected sample should be chosen so that 18% of the people in the sample are under 20.

Suggested applications

- Prepare questionnaires and discuss the consistency of presentation and possible different interpretations of the questions. Teachers could briefly address the issues of non-response or unexpected response. (Questionnaires are often ‘piloted’ before being finalised.)
- Students could investigate the shuffle mode on an mp3 player as an example of random selection.
- The random number generator on a calculator could be used to generate random numbers, eg to randomly select a student from the class roll.

DS2 Displaying and interpreting single data sets

The principal focus of this topic is to have students gain experience in preparing a variety of data displays. The power of statistical displays, both to inform and to misinform, should be emphasised.

Outcomes addressed

MGP-2, MGP-5, MGP-7, MGP-9, MGP-10

Content

Students:

- construct a dot plot from a small data set and interpret the dot plot
- draw a radar chart to display data
- create statistical displays using a spreadsheet or other appropriate software
- divide large sets of data into deciles, quartiles and percentiles and interpret displays
- calculate and interpret the range and interquartile range as measures of the spread of a data set
- construct frequency tables for grouped data from cumulative frequency graphs (histograms and polygons)
- estimate the median and upper and lower quartiles of a data set from a cumulative frequency polygon for grouped data
- establish a five-number summary for a data set (lower extreme, lower quartile, median, upper quartile and upper extreme)
- determine the five-number summary from a stem-and-leaf plot
- develop a box-and-whisker plot from a five-number summary
- link type of data with an appropriate display, eg continuous quantitative data with a histogram, or categorical data with a divided bar graph or sector graph (pie chart)
- interpret the various displays of single data sets
- identify the misrepresentation of data.

Considerations

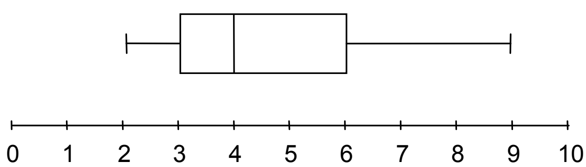
- Teachers may find it necessary to revise (i) the construction of tally charts and frequency tables for ungrouped and grouped data and (ii) the appropriate selection of dot plots, sector graphs, bar graphs, stem-and-leaf plots, histograms or line graphs to display data sets. They may also find it necessary to revise the selection of appropriate scales.
- Examples of data that could be displayed as a radar chart include sales figures, temperature readings and rainfall readings.
- Examples of readily available percentile charts include Infant Length for Age percentile charts.
- A clear distinction should be made between histograms and column graphs or bar graphs, particularly when using spreadsheets to generate graphical representations. A histogram is a graphical display of tabulated frequencies. It shows the proportion of cases that fall into each of several class intervals (also called bins). In a histogram, a value is denoted by the area of the bar rather than the height of the bar, as in a regular column graph or bar graph. This difference is important when the class intervals are not of uniform width. In a histogram, the class intervals do not overlap and must be adjacent. A histogram requires a continuous scale on the horizontal axis, with the frequency plotted on the vertical axis. A histogram is usually a display of continuous data.

  The width chosen for the bins determines the number of bins. It should be noted that there is no single best number of bins and that different bin sizes can reveal different aspects of the data. It is useful to experiment with the bin width in order to best illustrate important features of the data.
- Students could be given opportunities to identify outlying values in a data set at this stage and to suggest possible reasons for their occurrence. (Formal definition of outlying values is not intended in the Preliminary Mathematics General course.)
- The suitability of different types of statistical displays should be compared, eg a radar chart can be used to illustrate change in a quantity over time, especially for cyclical data (daily, seasonal, annual, etc), while a line graph is useful to show trends in data over equal time intervals.
- *For students intending to proceed to the HSC Mathematics General 2 course, teachers should consider introducing, at this stage, examples where the size of the class interval for grouped data is varied, and discussing the effect that this may have on the histogram and its interpretation. This can be facilitated through the use of appropriate technology.*

Suggested applications

- Students could collect examples of misleading statistical displays and prepare accurate versions of each. They could describe the inaccuracies, and their corrected version, in appropriate mathematical language.
- Using the box-and-whisker plot, what percentage of drivers in this sample have reaction times of 3 or more seconds? What percentage of the drivers have reaction times between 4 and 9 seconds? What is the interquartile range for this data set?

Reaction time in seconds prior to braking − drivers over 55



DS3 Summary statistics

The principal focus of this topic is the calculation of summary statistics for single data sets and their use in interpretation. Students gain experience with both ungrouped and grouped data.

Outcomes addressed

MGP-1, MGP-2, MGP-7, MGP-9, MGP-10

Content

Students:

- calculate the median, including from stem-and-leaf plots and cumulative frequency polygons
- calculate the measures of location – mean, mode and median – for grouped data presented in table or graphical form
- determine the mean for larger data sets of either ungrouped or grouped data using the statistical functions of a calculator
- describe standard deviation informally as a measure of the spread of data in relation to the mean
- calculate standard deviation using a calculator
- calculate summary statistics using spreadsheet formulae
- select and use the appropriate statistic (mean, median or mode) to describe features of a data set, eg median house price or modal shirt size
- compare summary statistics of various samples from the same population
- assess the effect of outlying values on summary statistics for small data sets.

Considerations

- Teachers may find it necessary to revise calculation of the mean for *small* data sets, using the formula

$$\bar{x} = \frac{sum\ of\ scores}{number\ of\ scores}$$

- Teachers should demonstrate the effect of changing value(s) in a data set on the summary statistics. Outlying values could be investigated in this way.
- Class intervals should be restricted to equal intervals only.
- For the Preliminary Mathematics General course, the HSC Mathematics General 2 course and the HSC Mathematics General 1 course, the standard deviation is taken to be the population standard deviation.
- *For students intending to proceed to the HSC Mathematics General 2 course, teachers should consider introducing, at this stage, examples where summary statistics for two sets of data need to be compared.*

Suggested applications

- Interpret and evaluate data from students' own data sets and draw conclusions that can be justified.
- Use a spreadsheet to examine the effect on the calculated summary statistics of changing the value of a score. The spreadsheet below provides such an example. There is only one difference between the two sets of data: for the fifth student in Set 2, the outlying value of 201 cm has the effect of increasing the mean and standard deviation, while leaving the median unchanged.

	A	B	C	D	E	F
1	Set 1			Set 2		
2	Student	Heights (cm)		Student	Heights (cm)	
3	1	146		1	146	
4	2	162		2	162	
5	3	167		3	167	
6	4	168		4	168	
7	5	170		5	201	
8						
9	Mean:	162.6		Mean:	168.8	
10	Median:	167		Median:	167	
11	S.D.:	8.71		S.D.:	17.93	
12						
13						

- Use the following spreadsheet functions on a range of cells containing numerical data to calculate summary statistics: sum, minimum, maximum, mean, mode, median, quartile and (population) standard deviation.

Strand: Measurement

Measurement involves the application of knowledge, skills and understanding in number and geometry in quantifying, and in solving problems in practical situations. The study of Measurement is important in developing students' ability to make reasonable estimates for quantities, to apply appropriate levels of accuracy to particular situations, and to apply understanding of such aspects of measurement as length, area, volume, similarity, and trigonometry to a variety of problems. Trigonometry, for example, has a range of applications in such fields as surveying, navigation, meteorology, architecture, construction and electronics.

In the Measurement Strand in the Preliminary Mathematics General course, students investigate metric units of measurement and their applications, and solve a range of problems involving rates and ratios, perimeter, area and volume, the application of similarity properties, and right-angled triangles.

Outcomes addressed

A student:

MGP-2 represents information in symbolic, graphical and tabular form

MGP-3 represents the relationships between changing quantities in algebraic and graphical form

MGP-4 performs calculations in relation to two-dimensional and three-dimensional figures

MGP-5 demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units.

Content summary

MM1 Units of measurement and applications

MM2 Applications of perimeter, area and volume

MM3 Similarity of two-dimensional figures, right-angled triangles

Terminology

accuracy
adjacent
angle of depression
angle of elevation
area
capacity
composite figure
cosine ratio
cylinder
error
field diagram
hypotenuse
opposite
perimeter
quadrant
rate
ratio
right prism
scale diagram
scale factor
scientific notation
significant figures
similar
sine ratio
standard prefix
tangent ratio
trigonometry
unitary method
volume

Use of technology

Students should be given the opportunity to use suitable software applications to draw accurate figures, investigate geometrical properties, and create both two-dimensional (2D) and three-dimensional (3D) drawings. A spreadsheet could be used to investigate maximising the perimeters and areas of shapes.

Online conversion calculators are readily available and could be used to investigate conversion of units.

Online maps (eg as available through Google Earth) may be used to enhance visualisation of scale, to calculate distances, and to make estimations of travelling times. Such resources could also be used to make estimations and calculations of areas of land using field diagrams.

Notes

The Focus Studies provide many opportunities for students to apply skills developed in the Measurement Strand in the Preliminary Mathematics General course. Teachers can draw upon problems from a wide variety of sources to reinforce the skills developed. For example, work on units of measurement is revised and extended in the Preliminary Mathematics General course Focus Studies: Mathematics and Communication, and Mathematics and Driving. (Rates and ratios are further addressed, in context, in each of the HSC Mathematics General 1 course Focus Studies: Mathematics and Design, Mathematics and Household Finance, Mathematics and the Human Body, and Mathematics and Personal Resource Usage.)

Where possible, students should be given the opportunity to gain practical experiences in relation to the mathematics addressed in the topic areas. Where it is not possible to provide practical experiences, the problems posed should be relevant to the lives of students.

MM1 Units of measurement and applications

The principal focus of this topic is on metric units of measurement, and rates and ratios. Students learn to make judgements about measurement errors.

Outcomes addressed

MGP-2, MGP-3, MGP-4, MGP-5

Content

Students:

- repeat and average measurements to reduce the likelihood of error
- investigate the degree of accuracy of reported measurements, including the use of significant figures where appropriate
- use scientific notation and standard prefixes in the context of measurement
- express measurements in scientific notation
- calculate with ratios, including finding the ratio of two quantities, dividing quantities in a given ratio, and using the unitary method to solve problems
- calculate rates, including pay rates, rates of flow, and rates of speed
- convert between units for rates, eg km/h to m/s, mL/min to L/h
- convert between common units for area
- convert between common units for volume
- determine the overall change in a quantity following repeated percentage changes, eg an increase of 20% followed by a decrease of 20%.

Considerations

- Teachers may find it necessary to revise:
  - determining the appropriate units to use when measuring
  - conversion between commonly used units using standard prefixes
  - accuracy of physical measurement being limited to $\pm\frac{1}{2}$ of the smallest unit of which the measuring instrument is capable, including determination of possible sources of error in measuring.
- Standard prefixes need to include nano-, micro-, milli-, centi-, kilo-, mega-, giga- and tera-.
- Students should be able to make conversions between units for rates over both dimensions, eg length and time, including km/h to m/s.
- *For students intending to proceed to the HSC Mathematics General 2 course, teachers could at this stage:*
  - *calculate the percentage error in a measurement*

    eg if the measured height was 155 cm ± 0.5 cm (ie to the nearest centimetre), the percentage error for this measurement is $\pm\left(\frac{0.5}{155}\right)\times 100\%$ *.*

    Students could measure their heights and calculate the percentage error in the measurement.
  - *calculate concentrations expressed as weight/weight, weight/volume or volume/volume*

    eg a patient needs 3 litres of fluid per day. One millilitre of fluid contains approximately 15 drops. Find the rate at which the intravenous drip must run, expressing the answer in the form: number of drops fed to the patient per minute.

Suggested applications

- Modify given recipes by varying quantities to provide for various numbers of people.
- Calculate the quantity of each component needed for a fertilising operation, given the ratio of the components in the mixture.
- Calculate and compare freight costs for a variety of modes of transport.
- Calculate rates of application of chemicals used in agriculture, such as rates for pesticides and feed additives.
- Calculate distances and travelling times from maps.
- If it costs 15 cents for 1 kilowatt (1000 watts) for one hour, how much would it cost for a 2400-watt heater to be on from 5 pm to 11 pm?

MM2 Applications of perimeter, area and volume

The principal focus of this topic is the calculation and application of perimeter, area and volume in the solution of problems.

Outcomes addressed

MGP-2, MGP-3, MGP-4, MGP-5

Content

Students:

- calculate the perimeter of simple figures, including right-angled triangles, circles, semicircles and quadrants
- calculate the perimeter and area of simple composite figures consisting of two shapes, including semicircles and quadrants
- calculate the perimeter and area of irregularly shaped blocks of land using a field diagram
- identify and use the correct formula to solve practical area problems
- calculate the volume of right prisms and cylinders using appropriate formulae
- estimate areas and volumes
- convert between units of volume and capacity.

Considerations

- Teachers may find it necessary to revise area calculations for triangles, quadrilaterals and circles.
- Some problems will require the application of Pythagoras' theorem.
- Students will need to use their algebraic skills to find a missing dimension of a shape, given its perimeter, area or volume.

Suggested applications

- Survey an irregularly shaped block of land using offsets and use this information to calculate its approximate area.
- Use a tape measure and chalk to determine the number of bricks in 1 square metre of a brick wall.
- Discuss and report on possible sources of error, eg experimental, instrumental and constant error, when determining the number of bricks in 1 square metre of a brick wall. In the context of measurement, error does not indicate a mistake; it is the term used to refer to the precision of a measurement.
- Estimate the painted surface area of a classroom.
- Investigate the dimensions that maximise the area for a given shape and perimeter, such as in the design of playpens and stock paddocks.
- Students use practical methods to investigate units of volume and capacity, eg by pouring contents of a soft drink bottle into a rectangular prism.
- How many kilograms of chicken manure would be required to fertilise a football field 100 m $\times$ 50 m if it is required that 8 g/m^2 be applied?

MM3 Similarity of two-dimensional figures, right-angled triangles

The principal focus of this topic is on solving a range of practical problems involving the application of similarity properties and right-angled triangles.

Outcomes addressed

MGP-2, MGP-3, MGP-4, MGP-5

Content

Students:

- calculate measurements from scale diagrams
- calculate scale factors for similar figures
- use scale factor to solve problems involving similar figures
- recognise that the ratio of matching sides in similar right-angled triangles is constant for equal angles
- calculate sine, cosine and tangent ratios
- use trigonometric ratios to find an unknown side-length in a right-angled triangle, when the unknown side-length is in the numerator of the ratio to be used
- use trigonometric ratios to find the size of an unknown angle in a right-angled triangle, correct to the nearest degree
- calculate angles of elevation and depression, given the appropriate diagram
- determine whether an answer seems reasonable by considering proportions within the triangle under consideration
- solve practical problems using scale diagrams and factors, similarity and trigonometry.

Considerations

- Teachers may find it necessary to revise:
  - calculating scale factors of objects and images
  - applying Pythagoras' theorem to practical situations.
- Scale diagrams should include house plans and maps. Online maps are readily available (with measurement tools) for extension activities.
- Trigonometric ratios should be introduced using similarity properties of right-angled triangles.
- The level of precision of angles given in questions should be limited to degrees.
- When using the tangent ratio students need to consider which angle should be used to ensure that the unknown is in the numerator.

Suggested applications

- Calculate the scale factor of enlargements obtained using an overhead projector.

  Is there a relationship between the distance of the projector from the screen and the scale factor of the resulting projection?
- A sewer is required to have a fall of 1 in 40. How much deeper should one end be compared to the other in a 160-metre-long trench?
- By measuring the shadow thrown by a metre-rule, students use similarity and shadow lengths to find the height of tall objects, eg a tree or a flagpole.
- Students accurately construct a scaled floor plan of the classroom.
- Find ceiling heights from building plans.
- Use house plans to cost carpeting, tiling, painting rooms, etc.
- Use a grid over a free-form diagram to draw an enlargement or reduction.
- Use the properties of a gate's diagonals to determine if it is rectangular.
- Investigate the trigonometric ratios for angles of, say, $30°$, $45°$, $60°$ and $75°$ in a number of similar right-angled triangles.
- In groups, students use a clinometer to find the heights of school buildings, etc. Different groups compare and discuss their answers.

Strand: Probability

Probability is of significant importance in everyday life. It is concerned with drawing conclusions related to the likelihood or chance that an event will occur. The study of Probability is important in developing students' awareness of the broad range of applications of probability concepts in everyday life, including, for example, in relation to the reliability of products, and how probability assessments are made and used in decision-making.

In the Probability Strand in the Preliminary Mathematics General course, students gain further experience in the use of the language of probability and compare relative frequency and theoretical probability.

Outcomes addressed

A student:

MGP-2 represents information in symbolic, graphical and tabular form

MGP-8 performs simple calculations in relation to the likelihood of familiar events

MGP-10 justifies a response to a given problem using appropriate mathematical terminology.

Content summary

PB1 Relative frequency and probability

Terminology

complement
equally likely
event
experiment
experimental probability
favourable outcome
frequency
multistage experiment
outcome
probability
relative frequency
selection
theoretical probability

Use of technology

Students should make use of contextual data, as available on the internet, eg the Australian Bureau of Statistics (ABS) Yearbooks and the Australian Bureau of Meteorology are useful sources of data.

Students can create spreadsheets to record and display graphically the results of an experiment.

Notes

Students may have greater interest in probability contexts that relate to their life experiences, eg calculating the risk of injury as a probability for activities such as driving a car or playing contact sport.

Student-generated data, obtained from activities such as rolling dice and tossing coins, provide suitable data for analysis.

PB1 Relative frequency and probability

The principal focus of this topic is the counting of possible outcomes for an experiment or event, and comparing relative frequency and theoretical probability.

Outcomes addressed

MGP-2, MGP-8, MGP-10

Content

Students:

- determine the number of outcomes for a multistage experiment by multiplying the number of choices at each stage
- verify the total number of outcomes for simple multistage experiments by systematic listing
- comment critically on the validity of simple probability statements
- perform simple experiments and use recorded results to obtain relative frequencies
- estimate the relative frequencies of events from recorded data
- use relative frequencies to obtain approximate probabilities
- identify events with equally likely outcomes
- use the following definition of the theoretical probability of an event where outcomes are equally likely:

$$P(event) = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

- express probabilities as fractions, decimals and percentages
- recognise that $0 \le P(event) \le 1$
- compare theoretical probabilities with experimental estimates
- illustrate the results of experiments using statistical graphs and displays (see DS2 *Displaying and interpreting single data sets*)
- calculate the probability of the complement of an event using the relationship

$$P(an\ event\ does\ not\ occur) = 1 - P(the\ event\ does\ occur).$$

Considerations

- Factorial notation is not required in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course – the number of outcomes for a simple multistage experiment is determined by systematic listing.
- Statements involving the language of probability could be collected from various media (newspapers, magazines, radio, television, the internet, etc) and discussed.
- Data could be generated from simple experiments and also obtained from other sources, eg weather and sporting statistics from newspapers. Other data is available from Australian Bureau of Statistics (ABS) Yearbooks and various websites.
- Practical experiments could involve tossing coins, rolling dice, or selecting cards from a pack.

Suggested applications

- Comment critically on statements involving probability, such as: 'Since it either rains or is fine, the probability of a fine day is 50–50'.
- Investigate expressions used in other disciplines and in everyday life to describe likely or unlikely events, eg 'once in a blue moon', 'a one in 300-year flood', or 'a 75% chance of recovery following a medical operation'.
- Investigate the number of different meals that can be chosen from a menu.
- Determine the number of combinations of raised dots that are possible in the braille system for reading and writing. Investigate whether or not all the possible combinations are used. (Could undertake a similar activity for Morse code.)
- Investigate limitations on the number of postcodes or telephone numbers that can be used.
- Investigate the total number of possible car number plates available, given selected styles of number plates, eg two letters – two digits – two letters.
- Determine the number of possible computer passwords that could be generated from four digits. Compare the number of possible passwords if three digits and a letter are used. What effect does case sensitivity have on the number of passwords that contain letters?
- Experiments could be carried out in which the probability is not intuitively obvious, eg the probability of a drawing pin landing point up.
- Students could investigate the relative frequency of each different-coloured lolly when selecting a lolly from a packet of lollies of various colours.
- Examine the birth notices on a particular day in a major daily newspaper. Record the number of boys and the number of girls. On this basis, estimate the probability that a child born is (a) male or (b) female. Compare these results with those published by the Australian Bureau of Statistics (ABS).
- In a collection of DVDs, five are rated 'PG', three are rated 'G', and two are rated 'M'. If a DVD is selected at random from the collection, what is the probability that it is not rated 'M'?

Strand: Algebra and Modelling

Algebra involves the use of symbols to represent numbers or quantities and to express relationships. A mathematical model is a mathematical representation of a situation. All applications of mathematics are based on mathematical models. The study of Algebra and Modelling is important in developing students' reasoning skills and their ability to represent and solve problems.

In the Algebra and Modelling Strand in the Preliminary Mathematics General course, students apply algebraic skills and techniques to interpret and use simple mathematical models of real situations.

Outcomes addressed

A student:

MGP-1	uses mathematics and statistics to compare alternative solutions to contextual problems
MGP-2	represents information in symbolic, graphical and tabular form
MGP-3	represents the relationships between changing quantities in algebraic and graphical form
MGP-9	uses appropriate technology to organise information from a limited range of practical and everyday contexts
MGP-10	justifies a response to a given problem using appropriate mathematical terminology.

Content summary

AM1	Algebraic manipulation
AM2	Interpreting linear relationships

Terminology

algebraic expression	expand	simplify
axis	formulae	simultaneous linear equations
common difference	function	solution
constant	gradient	solve
conversion	independent variable	stepwise linear function
dependent variable	intercept	substitute
equation	linear	y-intercept
evaluate	point of intersection	

Use of technology

Students will require access to appropriate technology to create graphs of functions and to observe the effect on the graph of a function when parameters are changed.

Suitable graphing software should be used in investigating the similarities and differences between the graphs of a variety of linear relationships.

Students could use a spreadsheet to generate a table of values and the associated graph.

Notes

Algebraic skills should be developed through practical and vocational contexts.

Application of the skills developed in this Strand in the Preliminary Mathematics General course is consolidated further in Focus Studies in this (preliminary) course and in the HSC Mathematics General 2 course and the HSC Mathematics General 1 course.

Students should develop an understanding of a function as input, processing, output. (It is not intended that students learn a formal definition of a function.)

AM1 Algebraic manipulation

The principal focus of this topic is to provide a foundation in basic algebraic skills, including the solution of a variety of equations. The topic develops techniques that have applications in work-related and everyday contexts.

Outcomes addressed

MGP-1, MGP-2, MGP-3, MGP-9, MGP-10

Content

Students:

- add, subtract, multiply and divide algebraic terms
- simplify algebraic expressions involving multiplication and division,

  eg $\frac{9y}{4} \times 5y$, $\frac{4m}{5n} \div \frac{m}{20n}$, $\frac{4wn}{b} \times \frac{nb}{2w}$
- expand and simplify algebraic expressions
- substitute numerical values into algebraic expressions,
  eg $\frac{3x}{5}$, $5(2x-4)$, $3a^2-b$, $\sqrt{\frac{5p}{4m}}$, $\frac{x+y}{yx}$
- substitute given values for the other pronumerals in a mathematical formula from a vocational or other context to find the value of the subject of the formula,
  eg if $A = P(1+r)^n$, find A given $P = 600$, $r = 0.05$, $n = 3$
- solve linear equations involving two steps, eg $5x+12=22$, $\frac{4x}{10}=3$, $\frac{x-1}{3}=6$, $\frac{r}{5}-3=2$.

Considerations

- Emphasis should be placed on formulae from vocational and other practical contexts, including, but not limited to, formulae students will encounter in other topics.
- Substitution into expressions should include substitution into expressions containing multiple variables, positive and negative values, powers and square roots.
- *For students intending to proceed to the HSC Mathematics General 2 course, teachers could include at this stage:*
  - *addition and subtraction of simple algebraic fractions with different denominators, eg* $\frac{x}{6}+\frac{x}{4}$, $\frac{7x}{10}-\frac{x}{3}$
  - *factorisation of algebraic expressions, eg* $3p-6$, p^2+7p, $-2x-14$.

Suggested applications

- Students use formulae such as those below to find the values of pronumerals following substitution:

$C=\frac{5}{9}(F-32)$	$v=\frac{4}{3}\pi r^3$	$S=\frac{D}{T}$
$v^2=u^2+2as$	$S=V_0-Dn$	$V=IR$
$A=P(1+r)^n$	$h^2=a^2+b^2$	$P=I^2R$
$BAC_{Female}=\frac{10N-7.5H}{5.5M}$	$W=Fs$	$v=u+at$
(BAC: blood alcohol content)	$S=V_0(1-r)^n$	

AM2 Interpreting linear relationships

The principal focus of this topic is the graphing and interpretation of linear relationships in practical contexts.

Outcomes addressed

MGP-1, MGP-2, MGP-3, MGP-9, MGP-10

Content

Students:

- generate tables of values from a linear equation
- graph linear functions with pencil and paper, and with technology, given an equation or a table of values
- calculate the gradient of a straight line from a graph
- determine the y-intercept for a given graph
- identify independent and dependent variables in practical contexts
- establish a meaning for the intercept on the vertical axis in a given context
- sketch graphs of linear functions expressed in the form $y = mx + b$ without the use of tables
- sketch the graphs of a pair of linear equations to find the point of intersection
- find the solution of a pair of simultaneous linear equations from a given graph
- solve practical problems using graphs of simultaneous linear equations
- use stepwise linear functions to model and interpret practical situations, eg parking charges, taxi fares, tax payments and freight charges
- use graphs to make conversions, eg Australian dollars to euros
- use linear equations to model practical situations, eg simple interest
- describe the limitations of linear models in practical contexts.

Considerations

- The generalisation of linear number patterns should include the use of algebraic symbols to create an equation that describes the pattern. Terminology such as common difference and constant should be considered, as this will become useful in modelling linear relationships. Some time should be spent exploring tables of values that allow such discussion, eg

x	0	1	2	3
y	7	14	21	28

x	2	3	4	5
y	–3	–1	1	3

  Discussion should include:
  - the limitations of this method
  - that the common difference may be positive or negative
  - various methods for calculating the constant.
- Students should determine the gradient (or slope) of a straight line by forming a right-angled triangle.
- Graphs of straight lines in the form $y = mx + b$ could be sketched by first constructing a table of values, or by using appropriate technology and recognising and recording the important features, or by using some other method.
- Students should recognise the limitations of linear models in practical contexts, eg a person's height as a function of age may be approximated by a straight line for a limited number of years. Students should be aware that models may apply only over a particular domain.
- Technology should be used to construct graphical representations of algebraic expressions.
- *For students intending to proceed to the HSC Mathematics General 2 course, teachers could introduce at this stage:*
  - *development of a linear graph of the form $y = mx$ from a description of a situation in which one quantity varies directly with another*
  - *using the above graph to establish the value of m (the gradient) and to solve problems related to the given variation context*
  - *interpreting linear functions as models of physical phenomena*
  - *the meaning of the gradient and the y-intercept in various practical contexts.*

Suggested applications

- Students may test theories about how large or small the right-angled triangle should be to determine the gradient of a straight line. The sign of the gradient should be determined as positive or negative by inspecting whether it is 'uphill' or 'downhill' when looking from left to right.

Focus Study: Mathematics and Communication

Communication involves the interchange of information. In recent years, there has been a significant reduction in the costs of common forms of communication, and significant expansion in the power of communication devices such as computers, smartphones and tablets, as well as in internet and digital broadcasting and transmission technologies. It is important in modern society for consumers to maintain an ongoing awareness of the impact of developments in popular forms of communication.

In the Mathematics and Communication Focus Study, students apply, and develop further, knowledge, skills and understanding in Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling to practical communication contexts involving mobile phone plans, digital download/upload, and file storage.

Outcomes addressed

A student:

MGP-1	uses mathematics and statistics to compare alternative solutions to contextual problems
MGP-2	represents information in symbolic, graphical and tabular form
MGP-3	represents the relationships between changing quantities in algebraic and graphical form
MGP-5	demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
MGP-6	models financial situations relevant to the student's current life using appropriate tools
MGP-7	determines an appropriate form of organisation and representation of collected data
MGP-8	performs simple calculations in relation to the likelihood of familiar events
MGP-9	uses appropriate technology to organise information from a limited range of practical and everyday contexts
MGP-10	justifies a response to a given problem using appropriate mathematical terminology.

Content summary

FSCo1	Mobile phone plans
FSCo2	Digital download and file storage

Terminology

bits per second (bps)	download	megabyte (MB)
byte	excess data usage	pre-paid plan
cap plan	gigabyte (GB)	post-paid plan
connection fee	kilobits per second (kbps)	terabyte (TB)
data allowance	kilobyte (kB)	upload

Use of technology

Spreadsheets should be used to create graphs and calculate summary statistics.

The internet should be used as a source of up-to-date information, eg different mobile phone plans, including terms and conditions and charges.

Calculators can also be accessed on the internet for the calculation of the download/upload time for files.

Notes

Learning and teaching, and assessment, should draw on and integrate the mathematics in the Strands within the Preliminary Mathematics General course, ie Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling.

Students require access to current information from a range of sources, including mobile phone plans and mobile phone bills.

Students should develop the ability to extract and evaluate the underlying mathematics in a range of texts, eg mobile phone plan advertisements, comparison websites, and newspaper articles.

FSCo1 Mobile phone plans

The principal focus of this topic is the interpretation and comparison of mobile phone plans and usage, and the calculation of related costs.

Outcomes addressed

MGP-2, MGP-3, MGP-5, MGP-6, MGP-7, MGP-9, MGP-10

Content

Students:

- read and interpret mobile phone bills
- read and interpret mobile phone plans
- calculate the cost of calls, given the time and duration, based on different mobile phone plans
- calculate the cost of sending and receiving messages, given the details of the mobile phone plan
- calculate the cost of data usage, given the details of the mobile phone plan
- construct and interpret tables and graphs of mobile phone usage and the cost of making mobile phone calls (graphs should include step-wise linear functions)
- investigate patterns of usage for a given mobile phone bill
- determine a suitable mobile phone plan using calculations based on a typical usage pattern.

Considerations

- Students require access to sample mobile phone bills and current mobile phone plans from telecommunications providers.
- Students are to calculate the charges on a mobile phone bill given the call rates, connection fee, times calls are made, duration of calls, message rates, any excess data usage fees incurred, and terms and conditions of the mobile phone plan. Many mobile phone plans calculate call charges in set blocks of time, or part thereof, eg 30-second blocks. In the same way, mobile phone plans calculate excess data usage fees in set blocks of data or part thereof, eg 1 MB blocks. Students need to determine the number of chargeable blocks of time/data, based on the call duration/data usage.
- The cheapest minimum monthly plan may not always be the most cost-effective, given a particular person's phone usage. A comparison of the total cost for the same usage for different plans will demonstrate this.
- A 'cap plan' is not actually 'capped' so that charges cannot exceed the cap amount. A 'cap' plan consists of a predetermined amount, usually for a specified value of calls/messages/data, with any usage above this predetermined amount incurring additional charges.
- Graphs should be constructed using both pen and paper methods and technology.
- With teacher direction, students should brainstorm a list of the attributes of a mobile phone user who is making effective use of a mobile phone plan. Some of these attributes are:
  - knowing and understanding the plan
  - maximising use of free-time and off-peak rates
  - keeping usage within the allowances of the plan.

Suggested applications

- Students calculate excess usage charges and consider initial setup costs and fees and/or the costs of switching from one provider to another.
- Calculate the cost of a call of duration 3 minutes and 35 seconds, given that there is a connection fee of 40 cents and the call rate is 45 cents per 30-second block, or part thereof.
- Students could analyse their own mobile phone plans and usage, or analyse other examples of mobile phone plans that they have found, which include details of the rates and charges.
- Students describe the difference between a pre-paid plan, a post-paid plan and a cap plan.
- Students construct a variety of graph types, including those prescribed in DS2 *Displaying and interpreting single data sets* and stepwise linear function graphs. Radar charts are useful displays for data that is time-based, eg to display the monthly amount paid over one year.
- To identify usage patterns, students create a frequency table of the number of mobile phone calls made at different times of the day for a two-monthly bill. A radar chart could be constructed from this table. This exercise could be extended further by tabulating and graphing such variables as call duration and type of call, eg to a mobile phone and to a landline phone.

Call Time	Tally	Frequency
6 am to 7 am	||||	4
7 am to 8 am	|||	3
8 am to 9 am	|||	3
Etc		

- Students evaluate a person's typical mobile phone usage against several alternative plans in order to determine which of the plans is the most cost-effective for that person. This should include analysis of messaging, calling and data usage.

FSCo2 Digital download and file storage

In this topic, students perform a range of calculations, and collect and display data and interpret statistics, in relation to the download and storage of electronic files.

Outcomes addressed

MGP-1, MGP-2, MGP-3, MGP-5, MGP-6, MGP-7, MGP-8, MGP-9, MGP-10

Content

Students:

- use prefixes to describe the size of units of storage, eg mega-, giga-, tera-
- convert units of storage from bits to bytes, and vice versa
- convert between units for measuring memory size, file size, and secondary storage on devices such as USB drives and external hard drives
- calculate the time to download or upload a file, given a download or upload speed in bits per second, or kilobits per second, where a kilobit is defined as 1000 bits
- collect, display and analyse data on the downloading of music and videos
- calculate the probabilities of songs being played using the random selection mode
- interpret statistics related to the effect of downloading audio files and video files, legally and illegally, on the sales of media companies.

Considerations

- It should be noted that for downloading, transfer rates are expressed in bits per second, or kilobits per second (kbps), where 1 kilobit = 1000 bits.
- Units of memory size and file size could include, but not be limited to, bytes, kilobytes, megabytes and gigabytes. The following conversion factors apply:
  1 kilobyte = 2^{10} bytes = 1024 bytes
  1 megabyte = 2^{20} bytes = 1 048 576 bytes
  1 gigabyte = 2^{30} bytes = 1 073 741 824 bytes
  1 terabyte = 2^{40} bytes = 1 099 511 627 776 bytes

  Note: (i) In the conversion of units from bits to bytes, and vice versa, 1 byte = 8 bits.
  (ii) In this course, the kilobyte is taken to be synonymous with the kibibyte.
  (iii) For most computer operating systems, the storage capacity of a hard disk is calculated using the conversion factors stated above.
- Students should also be able to solve some more difficult questions, eg How many seconds would it take to download an 8.3 MB file if the transfer rate is 5 kbps?
- Data on the downloading of music and videos may, for example, include data on the quantity of downloads or on downloads categorised by age, gender, region and musical genre. See DS2 *Displaying and interpreting single data sets* and DS3 *Summary statistics* for knowledge, skills and understanding that could be applied.

Suggested applications

- As an extension activity, students could consider the costs and calculations associated with integrated plans that bundle landline services, mobile phones, and internet access.
- Determine the number of files of a particular format that can be stored on storage devices such as mp3 players, USB drives, and hard disk drives, eg calculate the number of mp3 files of average size 4.1 MB that can be stored on a 16 GB mp3 player.
- If an mp3 player contains five songs, in how many different ways could all five songs be played, with no song being repeated?
- A manufacturer claims that a 16 GB mp3 player holds 4000 songs.
  (i) Calculate the average file size of a typical song.
  (ii) Use calculations to either support or discount the manufacturer’s claim regarding the number of songs that the mp3 player will hold, eg collect file sizes for at least 50 songs and use a spreadsheet to calculate the average file size of a song.
- Apply mathematical reasoning to investigate the cost to the entertainment industry of illegal downloading of music and videos.

Focus Study: Mathematics and Driving

The use of personal motor vehicles is of significant importance to the lives of a high proportion of the population. The driving of such vehicles is first undertaken by the majority of people in later adolescence and continues throughout their adult lives. A strong awareness of the associated responsibilities and expenses is of fundamental importance in owning, maintaining and driving a motor vehicle. It is particularly important that this awareness is developed from the commencement of an individual's driving career, especially in relation to such safety matters as reaction times, braking distances and blood alcohol content.

In the Mathematics and Driving Focus Study, students apply, and develop further, knowledge and skills in Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling to practical contexts involving costs of purchase and insurance, running costs and depreciation, and the safety of motor vehicles.

Outcomes addressed

A student:

MGP-1	uses mathematics and statistics to compare alternative solutions to contextual problems
MGP-2	represents information in symbolic, graphical and tabular form
MGP-3	represents the relationships between changing quantities in algebraic and graphical form
MGP-5	demonstrates awareness of issues in practical measurement, including accuracy, and the choice of relevant units
MGP-6	models financial situations relevant to the student's current life using appropriate tools
MGP-7	determines an appropriate form of organisation and representation of collected data
MGP-8	performs simple calculations in relation to the likelihood of familiar events
MGP-9	uses appropriate technology to organise information from a limited range of practical and everyday contexts
MGP-10	justifies a response to a given problem using appropriate mathematical terminology.

Content summary

FSDr1	Costs of purchase and insurance
FSDr2	Running costs and depreciation
FSDr3	Safety

Terminology

alcohol consumption
blood alcohol content (BAC)
braking distance
comprehensive insurance
declining-balance method
depreciation
fuel consumption
reaction-time distance
salvage value
stamp duty
stopping distance
straight-line method
third-party insurance
trend
vehicle registration

Use of technology

Students should use spreadsheets to create graphical representations of collected data.

The internet should be used as a source of up-to-date information, eg current values of vehicles, stamp-duty rates, fuel prices, and interest rates for personal loans.

Online calculators can be used to investigate loan repayments.

Notes

Learning and teaching, and assessment, should draw upon and integrate the mathematics in the Strands within the Preliminary Mathematics General course, ie Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling.

Students require access to current information from a range of sources, including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

FSDr1 Costs of purchase and insurance

The principal focus of this topic is the calculation of motor vehicle purchase and insurance costs, and the interpretation of related tables and graphs.

Outcomes addressed

MGP-1, MGP-2, MGP-5, MGP-6, MGP-7, MGP-8, MGP-9

Content

Students:

- calculate the percentage decrease in the value of a new vehicle after one year
- describe the different types of insurance available, including compulsory and non-compulsory third-party insurance, and comprehensive insurance
- compare regional theft statistics and the related cost of insurance
- analyse theft and accident statistics in relation to insurance costs
- calculate the cost of stamp duty payable using current rates

  Note: this is a stepwise linear function
- create a line graph showing the stamp duty payable on vehicles of various prices, eg create a line graph for vehicles priced from \$1000 to \$80 000

  Note: this is a stepwise linear function
- compare the cost of purchase of different motor vehicles (cars and motorcycles only), including finance, transfer of registration, and insurance
- determine the monthly repayments on a reducing balance personal loan using tables or an online calculator
- compare the sale price of a car and the total amount repaid over the period of a loan.

Considerations

- Teachers should be sensitive to the situations and experiences of students when discussing accident statistics.
- Current interest rates for various lending institutions should be compared.
- Car loans are personal loans with a fixed monthly repayment. These loans are reducing balance loans and may be paid off in less time than the original term by increasing the monthly repayments, or by making additional payments.
- Stamp duty is levied by the Office of State Revenue when a vehicle is registered to a new owner. Stamp duty is paid on the market value of the vehicle or the price actually paid, whichever is greater.

  The stamp duty for a vehicle with a market value of \$50 000 was calculated in NSW in 2012 as follows: 3% of the market value of a vehicle up to \$45 000, plus 5% of the value of the vehicle over \$45 000:

Stamp duty on \$45 000 at 3%	= \$1350
Stamp duty on \$5000 at 5%	= \$250
Total stamp duty	= \$1600.

Suggested applications

- Students make decisions about the most appropriate way to display data comparing new models, and one-year-old models, of cars.

  Further discussions and investigations could include:

  – are the results similar for all makes of car?

  – are the results similar for motorcycles?

  Students also pose and investigate questions generated in classroom discussions.
- Given a set amount of money, students investigate the purchase of a vehicle and write a report. Research should include selecting a vehicle, making calculations if additional funding is required, the type of lending institution and lending rate, the amount payable in stamp duty, the cost of transferring registration fees, and insurance costs.
- Students investigate and write a report on the factors that affect insurance premiums, eg the type of vehicle, the age of the driver, and where the vehicle is to be garaged. Statistics related to vehicle theft, and the gender and age of drivers in accidents, should also be examined when investigating the cost of insurance and making comparisons.
- Students could construct a map to display the cost of insurance in different locations. Theft statistics could also be gathered and a comparison made between the cost of insurance and relevant theft statistics.
- Compare the cost of insurance for different makes of cars of the same size, or for the same make of car with different-aged drivers, or for the same make of car garaged at different addresses.

FSDr2 Running costs and depreciation

In this topic, students perform calculations in relation to motor vehicle fuel consumption and depreciation, and create and interpret related tables and graphs.

Outcomes addressed

MGP-1, MGP-2, MGP-6, MGP-7, MGP-9

Content

Students:

- identify fuel consumption measures as rates
- calculate the amount of fuel used on a trip
- compare fuel consumption statistics for various vehicles
- compare the amount of fuel needed and associated costs for various sizes, makes and models of vehicles, over various distances
- collect and present data on the price of fuel over time to identify trends
- calculate the depreciation of a vehicle using the straight-line method and the declining-balance method:
  (i) the straight-line method
  $S = V_0 - Dn$, where S is the salvage value of asset after n periods, V_0 is the initial value of the asset, D is the amount of depreciation per period, and n is the number of periods
  (ii) the declining-balance method
  $S = V_0 \left(1 - r\right)^n$, where S is the salvage value of asset after n periods, V_0 is the initial value of the asset, and r is the depreciation rate per period
- create a depreciation graph based on the straight-line method of depreciation (graphs to be produced from formulae and tables)
- use prepared graphs and tables of straight-line depreciation and declining-balance depreciation to solve problems.

Considerations

- Students should make calculations and comparisons of running costs using information from a table or graph.

Suggested applications

- Use online calculators to estimate the total weekly running costs for different makes and models of cars and motorcycles.
- Calculate and compare running costs for similar vehicles using different types of fuel, eg calculate and compare the running costs of a particular vehicle using petrol, diesel, or liquefied petroleum gas (LPG).
- Students could investigate the 'break-even' cost of installing LPG. A graph would be an appropriate method for displaying the results.
- Trends in fuel prices could be compared for different types of fuel and different locations.
- Calculate the yearly fuel consumption and the yearly cost of petrol for various classes of vehicle, eg a car with a 4 cylinder, 1.6 L engine compared to one with a 6 cylinder, 4 L engine, given their fuel consumption rate in litres per 100 kilometres.
- Investigate cycles in the price of unleaded petrol (ULP) and evaluate saving strategies over time.
- Compare country fuel prices to metropolitan fuel prices over time, and compare different fuel-type prices over time. Students can obtain data from the internet and present the data graphically.
- Calculate and compare the amount of depreciation of motor vehicles for different ages of the vehicles. (The depreciation in the first year of a new car can exceed 35%. For many vehicles, depreciation levels out to between 7% and 10% per annum after the first three years.)
- Students could compare the cost of an individual driving to and from a workplace to carpooling, car sharing and public transport for the same purpose.
- Actual running costs could be calculated from a logbook, which includes date, location, cost and amount of petrol purchased, and odometer reading.
- Students could investigate the environmental cost of running petrol-based cars compared to cars using other fuel types.

FSDr3 Safety

In this topic, students construct and interpret tables and graphs and solve a range of problems related to the safe operation of motor vehicles.

Outcomes addressed

MGP-1, MGP-2, MGP-3, MGP-7, MGP-9, MGP-10

Content

Students:

- calculate distance, speed and time, given two of the three quantities (with change of units of measurement as required), using

$$D = ST \qquad S = \frac{D}{T} \qquad T = \frac{D}{S}$$

- calculate stopping distance, including by substitution of values into suitable formulae
- construct and interpret tables and graphs relating to motor vehicles and motor vehicle accidents
- calculate and interpret blood alcohol content (BAC) based on drink consumption and body mass, including:
  - using formulae, both in word form and algebraic form, to calculate an estimate for BAC
  - using tables and graphs to estimate BAC
  - determining the number of hours required for a person who stops consuming alcohol to reach zero BAC
  - describing limitations of methods of estimating BAC
- construct and interpret graphs that illustrate the level of blood alcohol over time
- collect, represent and interpret data relating to driver behaviour and accident statistics.

Considerations

- The average speed of a journey is calculated using the formula

$$average\ speed = \frac{total\ distance\ travelled}{total\ time\ taken}$$

- Stopping distance can be calculated using the formula

  stopping distance = reaction-time distance + braking distance

- Reaction time is the time period from when a driver decides to brake to when the driver first commences braking.
- Students should investigate the meaning of 'standard drink'.
- Students should discuss why blood alcohol content (BAC) is a function of body weight.
- Formulae used for estimating BAC include:

  $$BAC_{Male} = \frac{10N - 7.5H}{6.8M} \text{ and } BAC_{Female} = \frac{10N - 7.5H}{5.5M}$$

  where N is the number of standard drinks consumed, H is the number of hours of drinking, and M is the person's mass in kilograms.

  Note: the following are limitations to the estimation of BAC:

  – formulae and tables are based on average values and will not apply equally to everyone

  – factors (variables) that affect BAC include gender, weight, fitness, health, and liver function.

- Students should discuss the limitations stated above, including why BAC differs for males and females.
- Zero BAC is an important consideration for young drivers in NSW, as the state's laws require a zero BAC limit for all learner and provisional drivers.

Suggested applications

- Students could investigate and make comparisons of legal blood alcohol limits in different countries.
- Students could investigate:
  - the safety aspects of stopping distances in relation to speed limits, eg calculate by formula the difference in stopping distance if travelling 5 km/h over the speed limit
  - stopping distances for different speeds, road conditions, and weather conditions.

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10 HSC Mathematics General 2 Course Content

Hours shown are indicative only.

Strands — 80 hours

Financial Mathematics

FM4 Credit and borrowing
FM5 Annuities and loan repayments

Data and Statistics

DS4 Interpreting sets of data
DS5 The normal distribution
DS6 Sampling and populations

Measurement

MM4 Further applications of area and volume
MM5 Applications of trigonometry
MM6 Spherical geometry

Probability

PB2 Multistage events and applications of probability

Algebra and Modelling

AM3 Further algebraic skills and techniques
AM4 Modelling linear relationships
AM5 Modelling non-linear relationships

Focus Studies — 40 hours

Mathematics and Health

FSHe1 Body measurements
FSHe2 Medication
FSHe3 Life expectancy

Mathematics and Resources

FSRe1 Water availability and usage
FSRe2 Dams, land and catchment areas
FSRe3 Energy and sustainability

Total indicative hours — 120 hours

Strand: Financial Mathematics

A sound understanding of credit, the responsible use of, and costs associated with, credit cards, and borrowing and investing money are important in developing students' ability to make informed financial decisions. This includes a sound understanding of annuities, which represent fixed payments into an investment account or for the repayment of a loan.

In the Financial Mathematics Strand in the HSC Mathematics General 2 course, the principal focus is on the mathematics of borrowing money and making informed decisions about financial situations.

Outcomes addressed

A student:

MG2H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts
MG2H-3	makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions
MG2H-6	makes informed decisions about financial situations, including annuities and loan repayments
MG2H-9	chooses and uses appropriate technology to locate and organise information from a range of contexts
MG2H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

FM4	Credit and borrowing
FM5	Annuities and loan repayments

Terminology

annuity	future value	periodical contribution
annuity stream	future value of an annuity	present value
balance	initial amount	present value of an annuity
credit card	interest	principal
final amount	interest factor	reducing-balance loan
flat-rate loan	loan repayment table	repayment

Use of technology

Prepared spreadsheets can be used to simulate loans and to make related formulae easier to understand and implement.

Students calculate, graph and analyse loan repayments and can use the financial functions of a spreadsheet to solve problems involving annuities.

Students use spreadsheets or other appropriate software to construct tables and graphs to illustrate annuities.

Notes

In FM5 *Annuities and loan repayments,* calculations of annuities are to be performed using a table of interest factors. Calculation of annuities through the use of a table, rather than through the use of formulae, provides additional time for the development of an appropriate understanding of the underlying concepts.

Teachers and students should have access to actual financial information and products, eg examples of credit card statements should be used in learning and teaching.

Students should be familiar with correct terminology in the topic areas and be able to express themselves using this terminology when justifying or explaining their solutions to problems.

FM4 Credit and borrowing

This topic focuses on the mathematics involved in borrowing money, the different types of loans available, and credit cards.

Outcomes addressed

MG2H-1, MG2H-3, MG2H-9, MG2H-10

Content

Students:

- calculate credit card payments, interest charges, and balances
- calculate principal, interest and repayments for flat-rate loans
- calculate values in a table of loan repayments
- recognise that P, the principal (or initial amount) in calculations with the compound interest formula $A = P(1+r)^n$ is the 'present value' (PV) and A, the final amount, is the 'future value' (FV)
- calculate future value and present value using the formula

  $$FV = PV(1+r)^n$$

- compare different options for borrowing money in relation to total repayments, fees, interest rates.

Considerations

- Credit card payment calculations should involve fees and interest-free periods.
- Students should be able to calculate the next row, or a particular value for a row/column, in a table of loan repayments, eg calculate the next row (ie when $N = 5$) in the loan repayment table below.

Loan table				
	Amount = $50 000		This table assumes the same number of days in each month, ie Interest = Rate/12 x Principal	
	Annual interest rate = 10%			
	Monthly repayment (R) = $600			
N	Principal (P)	Interest (I)	$P + I$	$P + I - R$
1	$50 000	$416.67	$50 416.67	$49 816.67
2	$49 816.67	$415.14	$50 231.81	$49 631.81
3	$49 631.81	$413.60	$50 045.40	$49 445.40
4	$49 445.40	$412.05	$49 857.45	$49 257.45

- The 'compounded value' of a dollar is also known as the 'future value' of a dollar.
- In the financial world, the compound interest formula is known as the future value formula and is expressed as $FV = PV(1+r)^n$.

  The single amount that needs to be invested when the future value is known can be calculated using the formula $PV = \frac{FV}{(1+r)^n}$.

  Note: this is the future value formula rearranged to make the present value the subject. This formula also applies to the present value of an annuity (see FM5 *Annuities and loan repayments*).

Suggested applications

- Plan a spreadsheet for a reducing-balance loan using paper, pen and calculator, and then construct the spreadsheet. Consider car loans, travel loans, loans for capital items, and home loans, as well as other types of loans.
- Students use a loan spreadsheet or an online simulator to vary the amount borrowed, the interest rate, and the repayment amount. They determine the answers to suitable 'what if' questions, eg 'What is the effect on the term of the loan and the amount of interest paid of (i) an interest rate rise, (ii) paying more than the minimum monthly repayment, and (iii) an interest rate rise in the case where more than the minimum monthly repayment is already being paid?'
- Use a prepared graph of 'amount outstanding' versus 'repayment period' to determine when a particular loan will be half-paid.

FM5 Annuities and loan repayments

The principal focus of this topic is the nature and mathematics of annuities, the processes by which they accrue, and ways of maximising their value as an investment. Annuity calculations are also used to calculate the present value of a series of payments and to calculate the repayment amount of a reducing-balance loan. Emphasis should be placed on using tables of interest factors to facilitate calculations.

Outcomes addressed

MG2H-1, MG2H-3, MG2H-6, MG2H-9, MG2H-10

Content

Students:

- recognise that an annuity is a financial plan involving periodical, equal contributions to an account, with interest compounding at the conclusion of each period
- calculate (i) the *future value of an annuity* (FVA) and (ii) the contribution per period, using a table of future value interest factors for calculating a single future value of an annuity stream
- recognise that the values in a table of future value interest factors can be obtained using the formula for the future value of an annuity
- calculate (i) the *present value of an annuity* (PVA) and (ii) the contribution per period, using a table of present value interest factors for calculating a single present value of an annuity stream
- recognise that the values in a table of present value interest factors can be obtained using the formula for the present value of an annuity
- use a table of interest factors for the present value of an annuity to calculate loan instalments, and hence the total amount paid over the term of a loan
- investigate the various processes for repayment of loans
- calculate the monthly repayment for a home loan from a table, given the principal, rate and term
- calculate the fees and charges that apply to different options for borrowing money in order to make a purchase
- interpret graphs that compare two or more repayment options for home loans.

Considerations

- Students will not be required to calculate the values of annuities using formulae.
- Teachers may wish to construct a table of future value interest factors for annuities and a table of present value interest factors for annuities using the appropriate formulae:

$$FVA = a\left\{\frac{(1+r)^n - 1}{r}\right\} \qquad PVA = a\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\},$$

where a is the contribution per period paid at the end of the period, r is the interest rate per compounding period, and n is the number of periods.

Suggested applications

- Students calculate the future value of an annuity (FVA) using a table:

Table of future value interest factors					
	Interest rate per period				
Period	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525

For example, using the table, the future value of an annuity of $\$1200$ per year for three years at 5% pa is $3.1525 \times \$1200 = \$3783$.

- Students use a table to calculate the present value of an annuity (PVA) or to calculate the contribution per period, eg to determine the monthly repayment in the case of a reducing-balance car loan.

Table of present value interest factors						
r	0.0060	0.0065	0.0070	**0.0075**	0.0080	0.0085
N						
45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383
46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133
47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311
48	41.59882	41.11986	40.64856	**40.18478**	39.72839	39.27924
49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975
50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470

In the example indicated in the table, the present value interest factor for a loan with interest rate 0.0075 per month, expressed as a decimal (or 9% pa), over 48 months (four years) is 40.18478.

The monthly repayment a for a car loan of $\$8000$ at 9% pa for four years is calculated as follows:

$$\$8000 = a \times 40.18478$$

$$a = \$8000 \div 40.18478 = \$199 \text{ to the nearest dollar.}$$

Strand: Data and Statistics

The collection and statistical analysis of data is of fundamental importance in society. Conclusions drawn from the statistical analysis of data are used to inform decisions made by governments and, for example, in science, business and industry. It is important that students have a broad understanding of how data and statistics are used, as well as how statistics are misused, resulting in the misrepresentation of information. Concepts related to 'normally distributed' data are important aspects of statistical analysis and are key to making informed decisions based on the analysis.

In the Data and Statistics Strand in the HSC Mathematics General 2 course, students interpret and compare sets of related data using summary statistics and graphical displays, apply basic concepts of normal distributions, and use samples to draw conclusions about populations from which the samples were drawn.

Outcomes addressed

A student:

MG2H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts
MG2H-2	analyses representations of data in order to make inferences, predictions and conclusions
MG2H-7	answers questions requiring statistical processes, including the use of the normal distribution, and the correlation of bivariate data
MG2H-8	solves problems involving counting techniques, multistage events and expectation
MG2H-9	chooses and uses appropriate technology to locate and organise information from a range of contexts
MG2H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

DS4	Interpreting sets of data
DS5	The normal distribution
DS6	Sampling and populations

Terminology

area chart
bell-shaped
box-and-whisker plot
capture/recapture technique
class interval
counting technique
cross-tabulation
cumulative frequency
data set
double stem-and-leaf plot
frequency table
grouped data
histogram
interquartile range
lower quartile
measure of location
measure of spread
multiple display
normal distribution
outlier
polygon
population characteristic
population mean
population standard deviation
quartile
radar chart
random number
range
sample mean
side-by-side multiple display
skewness
smoothness
standard deviation
standardised score
statistical method
stem-and-leaf plot
symmetry
two-way table
ungrouped data
upper quartile
z-score

Use of technology

Spreadsheets or other appropriate statistical software can be used to construct frequency tables and to calculate mean and standard deviation.

The effect of outliers on the mean, median and mode of a data set can be investigated using appropriate technology.

Notes

Students need access to real data sets and contexts. They can also develop their own data sets for analysis or use some of the data sets available online. Online data sources include the Australian Bureau of Statistics (ABS) website.

DS6 *Sampling and populations* requires the integration of knowledge, skills and understanding from Data and Statistics and from Probability.

Suitable data sets for statistical analysis could include, but are not limited to, home versus away sports scores, male versus female data (eg for height), young people versus older people data (eg for blood pressure), population pyramids of countries over time, customer waiting times at fast-food outlets at different times of the day, and monthly rainfall for different cities or regions.

DS4 Interpreting sets of data

The principal focus of this topic is the use of data displays, measures of location, and measures of spread to summarise and interpret one or more sets of data, in which the data is either ungrouped or grouped.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-7, MG2H-9, MG2H-10

Content

Students:

- represent large data sets as grouped data using frequency tables and histograms
- compare histograms for grouped data when varying the size of the class interval
- estimate measures of location, including median, upper and lower quartiles, from frequency tables, cumulative frequency tables, and cumulative frequency histograms and polygons
- calculate measures of location for grouped data: mean, mode and median
- calculate measures of spread: range, interquartile range, and population standard deviation
- calculate and make comparisons of the population standard deviations of two or more sets of data
- identify outliers in data sets and their effect on the mean, median and mode
- describe the general shape of a graph or display that represents a given data set, eg in terms of smoothness, symmetry, skewness or number of modes
- make judgements about data based on observed features of a display
- display data in double (back-to-back) stem-and-leaf plots
- display data in two box-and-whisker plots drawn on the same scale
- determine the percentages of data between any two quartiles on a box-and-whisker plot
- display two sets of data on a radar chart
- use side-by-side multiple displays of the same data set, eg a side-by-side histogram and a box-and-whisker plot
- prepare an area chart to illustrate and compare different sets of data over time
- use multiple displays to describe and interpret the relationships between data sets
- compare summary statistics, including mean, mode, median, and population standard deviation, for two sets of data
- interpret data presented in two-way table form, eg male/female versus exercise/no exercise
- group and compare variables within the same data set using cross-tabulation.

Considerations

- Class intervals should be restricted to equal intervals only.
- Interpretation of data sets includes deciding whether clustering is present, whether the shape of the display indicates any skewness of scores, if there are outliers, or if there is any other tendency in the data.
- In this course, an outlier is defined as a data point that is either:
  - less than $Q_L - 1.5 \times IQR$ or
  - greater than $Q_U + 1.5 \times IQR$,

  where Q_L is the lower quartile, Q_U is the upper quartile, and IQR is the interquartile range.
- Quartiles should be determined for data sets containing odd and even numbers of data values. In calculating the first and third quartiles, the median is excluded. Students should be aware that the second quartile is the median.
- Cross-tabulation of data includes grouping and comparing variables within the same data set. This process can be facilitated using the pivot table capability of a spreadsheet program.

Suggested applications

- Conduct a cross-tabular analysis by pivot table of heart rate data after exercise. A portion of such a data set (where $n = 30$) is shown below:

Height of step	**Stepping rate**	**Heart rate**
Short step	Slow	93
Tall step	Medium	111
Tall step	Fast	120
Short step	Fast	123
Short step	Medium	96
Tall step	Slow	99
Tall step	Slow	99
Tall step	Medium	129

Using a spreadsheet, the following pivot table was created for this data set:

		Stepping rate		
Height of step	Data	Slow	Medium	Fast
Short step	Average of heart rate	87.6	94.2	108.0
	Standard deviation of heart rate	9.1	8.9	16.7
Tall step	Average of heart rate	103.8	114.0	136.8
	Standard deviation of heart rate	10.5	21.0	13.3

What is the average heart rate for a short step at a fast stepping rate? Which combination of step height and stepping rate showed the greatest variability?

DS5 The normal distribution

In this topic, students apply properties of the standard normal distribution to the solution of real-life problems.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-7, MG2H-9, MG2H-10

Content

Students:

- describe the z-score (standardised score) corresponding to a particular score in a set of scores as a number indicating the position of that score relative to the mean
- use the formula $z = \frac{x - \overline{x}}{s}$ to calculate z-scores, where s is the standard deviation
- use calculated z-scores to compare scores from different data sets
- identify properties of data that are normally distributed, eg
  - the mean, median and mode are equal
  - if represented by a histogram, the resulting frequency graph is 'bell-shaped'
- use collected data to illustrate that for normally distributed data:
  - approximately 68% of scores will have z-scores between -1 and 1
  - approximately 95% of scores will have z-scores between -2 and 2
  - approximately 99.7% of scores will have z-scores between -3 and 3
- use these measures to make judgements in individual cases.

Considerations

- Graphical representations of data sets before and after standardisation should be explored.
- Teachers should compare two or more sets of scores before and after conversion to z-scores in order to assist explanation of the advantages of using standardised scores.
- Initially, students should explore z-scores from a pictorial perspective, investigating only whole number multiples of the standard deviation.
- Teachers should briefly explain the application of the normal distribution to quality control and the benefits to consumers of goods and services. Reference should be made to situations where quality control guidelines need to be very accurate, eg the manufacturing of medications.

Suggested applications

- Given the means and standard deviations of each set of test scores, compare student performances in the different tests to establish which is the 'better' performance.
- Packets of rice are each labelled as having a mass of 1 kg. The mass of these packets is normally distributed with a mean of 1.02 kg and a standard deviation of 0.01 kg. Complete the following table:

Mass in kg	1.00	1.01	1.02	1.03	1.04
z-score			0	1	

  (a) What percentage of packets will have a mass less than 1.02 kg?

  (b) What percentage of packets will have a mass between 1.00 and 1.04 kg?

  (c) What percentage of packets will have a mass between 1.00 and 1.02 kg?

  (d) What percentage of packets will have a mass less than the labelled mass?

- A machine is set for the production of cylinders of mean diameter 5.00 cm, with standard deviation 0.020 cm. Assuming a normal distribution, between what values will 95% of the diameters lie?

  If a cylinder, randomly selected from this production, has a diameter of 5.070 cm, what conclusion could be drawn?
- Given the frequency table for a familiar human characteristic, students could analyse the data to determine whether or not it could be considered normally distributed.
- Students could investigate whether the results of a particular experiment are normally distributed.

DS6 Sampling and populations

In this topic, students apply knowledge, skills and understanding developed in relation to counting techniques and statistical methods to select samples and draw conclusions about populations.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-7, MG2H-8, MG2H-9, MG2H-10

Content

Students:

- recognise that a sample can be used to provide an estimate for a particular population characteristic when the entire population cannot be assessed
- apply counting techniques to list all possible samples of varying sizes from a known small population (population sizes up to $n = 5$, sample sizes varying from $n = 1$ to $n = 5$)
- verify that the mean of the distribution of all possible sample means is equal to the population mean (μ) for populations (population sizes up to $n = 5$)
- describe and use the capture/recapture technique for estimating the size of populations, eg the number of fish in a lake
- generate random numbers with a table, calculator or spreadsheet to assist in establishing random samples
- recognise the effect of sample size in estimating the nature of a population, eg using the number of boys and girls in a particular Year 11 class to estimate the gender ratio in Year 11 across NSW.

Considerations

- Students should develop an understanding of the relationship between the distribution of the sample means and the population mean.
- The number in a known population, from which students list all possible samples of varying sizes, should be limited to five. This allows students to generate a list of all possible samples.
- It should be noted that 'without replacement' sampling avoids duplication that occurs when the same item is selected more than once. In large populations, it makes no difference to sample 'with replacement' or 'without replacement'. However, sampling 'with replacement' results in independent events that are unaffected by previous outcomes. Students could obtain and test results both 'with replacement' and 'without replacement'.
- Approximations of normal distributions will require investigation by graphing.
- This topic is not intended to be extended to calculations using the central limit theorem.

Suggested applications

- Students could use examples of population data, such as the number of family members in five different families or the different heights of basketball players in a team, for analysis.
- The capture/recapture technique can be simulated by using a large (but uncounted) number of toothpicks in a bowl to represent the population of fish in a particular waterway. (Students should discuss why it may not be practical to catch all of the fish in order to count them.) Take a small number (10 or 20) of the toothpicks and mark them with a coloured pen. Return them to the bowl and shake them around. This simulates the capture, tagging and release of a known number of fish. Now take a handful of the toothpicks from the bowl and establish the percentage of marked toothpicks (tagged fish) in the handful. Since the total number of tagged fish is known, this percentage may be used to estimate the size of the total population.
- Select a range of samples from a fixed population and record the characteristics of each sample.

Strand: Measurement

Perimeter, area, volume and trigonometry have many practical applications, including in building and surveying. Trigonometry provides various methods of obtaining unknown lengths, heights and distances and has applications in physics, chemistry, astronomy, navigation, and almost all branches of engineering. The study of the Earth as a sphere is important in developing students' understanding of distances and time differences between locations on the Earth.

In the Measurement Strand in the HSC Mathematics General 2 course, students extend and apply their knowledge, skills and understanding of perimeter, area and volume of more complex figures, and trigonometry. They solve a range of problems, including problems involving non-right-angled triangles and problems related to the Earth as a sphere.

Outcomes addressed

A student:

MG2H-4 analyses two-dimensional and three-dimensional models to solve practical problems, including those involving spheres and non-right-angled triangles

MG2H-5 interprets the results of measurements and calculations and makes judgements about reasonableness, including the degree of accuracy of measurements and calculations and the conversion to appropriate units

MG2H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

MM4 Further applications of area and volume

MM5 Applications of trigonometry

MM6 Spherical geometry

Terminology

angle of depression	great circle	rectangular pyramid
angle of elevation	Greenwich Meridian	right prism
annular cylinder	International Date Line	sector
annulus	latitude	Simpson’s rule
arc length	longitude	sine rule
closed cylinder	non-right-angled triangle	small circle
compass bearing	obtuse angle	sphere
compass radial survey	offset survey	square pyramid
composite solid	percentage error	surface area
cone	plane table survey	time zone
cosine rule	quadrant	trigonometric ratio
cylinder	radial survey	true bearing (three-figure bearing)

Use of technology

Teachers could use appropriate technology to provide students with local-area examples of aerial photographs for calculations, eg calculate the perimeter of a playing field or the area of an irregular-shaped field. Online distance calculators and a ruler could be used to determine the scale of a photograph.

Aerial photographs of lakes or dams, combined with research about their depths, could be used to make volume calculations.

Notes

Students should be encouraged to ‘estimate and check’ to determine if results are reasonable. This is a skill that should be reinforced throughout the Measurement Strand.

Application of the knowledge, skills and understanding developed in Measurement in the HSC Mathematics General 2 course is consolidated in the Focus Studies in the course.

Vocational applications of the concepts studied should be investigated, eg the use of Simpson’s rule in surveying.

MM4 Further applications of area and volume

The principal focus of this topic is to extend knowledge, skills and understanding developed in relation to perimeter, area and volume in the Preliminary Mathematics General course to include the surface area and volume of complex figures and the use of approximations in calculating the area and volume of irregular figures.

Outcomes addressed

MG2H-4, MG2H-5, MG2H-10

Content

Students:

- calculate the percentage error in a measurement,

  eg if the measured height was 155 cm ± 0.5 cm (ie to the nearest centimetre), the percentage error for this measurement is $\pm\left(\frac{0.5}{155}\right)\times 100\%$
- calculate areas of annuluses and parts of a circle (quadrant, sector), using appropriate formulae (area of annulus: $A = \pi(R^2 - r^2)$)
- calculate areas of composite figures constructed from squares, rectangles, triangles and circles
- apply Simpson's rule over three equally spaced points, ie one application (using $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$)
- calculate the surface area of right prisms
- calculate the surface area of cylinders (without 'top' and/or 'bottom') and closed cylinders ($Surface\ Area_{closed\ cylinder} = 2\pi\, r^2 + 2\pi\, rh$)
- calculate the surface area of spheres ($Surface\ Area_{sphere} = 4\pi\, r^2$)
- calculate the volume of a cone, square pyramid and rectangular pyramid using appropriate formulae ($Volume = \frac{1}{3}Ah$)
- calculate volumes of composite solids
- calculate the volume of an annular cylinder
- calculate the volume of right prisms, where the base is a composite or irregular two-dimensional shape, eg an I-beam
- determine errors in calculations resulting from errors made in measurement.

Considerations

- Manipulation of nets could be used to enhance understanding of the surface area of prisms, cylinders and pyramids.
- Students need to apply algebraic skills to find a missing dimension given the area, surface area or volume of a shape.
- Simpson's rule problems involving four strips should be treated through two applications of the rule. The problems addressed could be extended to include the calculation of volume given the relevant dimensions.

Suggested applications

- Design the shape and dimensions of a container that would have a particular capacity, given the purpose and use of the container.
- Design cost-effective packaging, eg groups of students are given four table-tennis balls and need to design two different boxes to package them. Students should then determine the better of the two designs in terms of minimisation of material used.
- As designers, students are given a square piece of metal of side length 2 metres from which to design an open rectangular water tank. The volume of water that the tank will hold depends on the size of squares cut from each of the four corners of the piece of metal. Students choose a scale, make models of tanks, and find the volume of water that they can hold. They could graph results and determine when the volume is the greatest. Students could also be asked to consider what happens if the side of the original square is doubled.
- Download a satellite map or other scale diagram from the internet and identify a geographical feature that has an irregular boundary, eg a lake or field. Use the scale of the map or diagram to calculate actual dimensions of the feature. Apply a formula such as Simpson's rule to estimate the area of the feature.
- Students calculate the surface area and volume of sphere-shaped sweets or cylinder-shaped sweets, eg marshmallows. Class results could be combined to form a data set for statistical analysis.

  Discussion could include:

  - What assumptions are being made about the shape of the sweet?
  - How can a reasonably accurate measure for the radius be obtained?
  - How accurate will the results be?
  - Are the results normally distributed?
  - What claims could the manufacturer make about the surface area and volume of that particular product?

MM5 Applications of trigonometry

The principal focus of this topic is to extend students' knowledge, skills and understanding in trigonometry to include trigonometry involving non-right-angled triangles. Problems to be solved include problems involving offset and radial surveys.

Outcomes addressed

MG2H-4, MG2H-5, MG2H-10

Content

Students:

- draw diagrams to represent information given about a right-angled triangle
- solve problems using trigonometric ratios in one or more right-angled triangles
- solve problems involving angles of elevation and depression, given the appropriate diagram
- establish the sine, cosine and tangent ratios for obtuse angles using a calculator
- determine the sign of the above ratios for obtuse angles
- use the sine rule to find side lengths and angles of triangles
- use the cosine rule to find side lengths and angles of triangles
- calculate the area of a triangle using the formula

$$A = \frac{1}{2}ab\sin C$$

- use appropriate trigonometric ratios and formulae in 'two-triangle problems', where one triangle is right-angled and the diagram is given
- use compass bearings (eight points only) and true bearings (three-figure bearings) in problem-solving related to maps and charts
- select and use appropriate trigonometric ratios and formulae to solve problems involving right-angled and non-right-angled triangles
- conduct radial (both plane table and compass) surveys
- solve problems involving non-right-angled triangle trigonometry, Pythagoras' theorem, and area in offset and radial surveys.

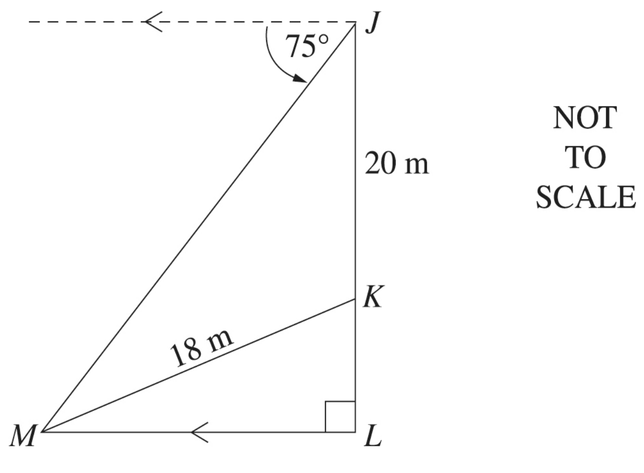
Considerations

- In this topic, the approximation of angles is to include approximation to the nearest minute.
- For problems involving application of the sine rule, it needs to be made clear to students whether an angle to be found is acute or obtuse.
- The cosine rule should be presented in each of the forms
  $c^2 = a^2 + b^2 - 2ab\cos C$ and $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.
- In problems that involve the sine rule, the cosine rule, or the area of a non-right-angled triangle, diagrams are to be provided.
- Diagrams should be provided for problems involving angles of elevation and depression, for problems involving bearings, and for problems involving two triangles.

Suggested applications

- Students estimate the area and perimeter of a part of the school grounds. They carry out a survey and complete a scale diagram, which they use to calculate the area and perimeter. Students present, compare and discuss their results. They are also asked to consider whether a different type of survey would give different results.
- Students could carry out a radial survey of an irregular area and compare the result with an offset survey of the same area.
- Plan a walk or another type of journey by reading and interpreting a map.
- Students could find gradients from contour lines on maps.
- Navigational charts could be used to plan routes and identify positions.
- Students could calculate the heights of buildings and other structures.
- The derivation of formulae used in this topic could be investigated.
- Examples of ‘two-triangle problems’ to be solved include:

  The angle of depression from J to M is $75°$. The length of JK is 20 m and the length of MK is 18 m.



Calculate the angle of elevation from M to K. Give your answer to the nearest degree.

MM6 Spherical geometry

The principal focus of this topic is to apply geometry and trigonometry to solve problems related to the Earth as a sphere. Applications include locating positions on the surface of the Earth using latitude and longitude, and calculating time differences.

Outcomes addressed

MG2H-4, MG2H-5, MG2H-10

Content

Students:

- calculate arc length of a circle using the formula

$$l = \frac{\theta}{360} 2\pi r$$

- distinguish between great and small circles
- use the equator and the Greenwich Meridian as lines of reference for locations on the Earth's surface
- locate positions on the surface of the Earth using latitude and longitude
- calculate distances, in kilometres, between two points on the same great circle
- use time zones and the International Date Line in solving problems
- calculate time differences between locations on the Earth given the difference in longitude
- determine the times in cities in different countries in travel questions.

Considerations

- For the calculation of time differences using longitude, apply $15° = 1$ hour time difference and $1° = 4$ minutes time difference. Daylight-saving time is to be considered.
- Teachers should note that it is not intended that nautical miles be used. The radius of the Earth is to be taken as 6400 km.
- Plotting longitude on a number line can provide a visual stimulus when calculating time differences. From a number line, students can readily determine whether to add or subtract time.

Suggested applications

- Calculate the present time in all the capital cities of Australia.
- For each of the states in Australia, investigate whether or not the state has daylight saving and, if so, when it starts and finishes.
- Students find three cities with the same time, for both a given number of hours earlier and a given number of hours later than local time. These could be marked on a world map and displayed to reinforce the concept of time zones.
- Find the coordinates of the point on the Earth's surface that is at maximum distance from the school's location.
- Examples of time-difference problems to be solved include:

  Cassie flew from London ($52°$N, $0°$E) to Manila ($15°$N, $120°$E). Her plane left London at $9{:}30$ am Monday (London time), stopped for five hours in Singapore, and arrived in Manila at 4 pm Tuesday (Manila time). What was the total flying time?

Strand: Probability

In developing an appreciation of the broad range of applications of probability in everyday life, students should be aware of its use in such fields as insurance and quality control in industry. The use of probability concepts also assists in making informed decisions in relation to games of chance.

In the Probability Strand in the HSC Mathematics General 2 course, students apply various techniques to multistaged events to calculate the probability of particular events occurring, as well as the probabilities of particular outcomes.

Outcomes addressed

A student:

MG2H-1 uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts

MG2H-2 analyses representations of data in order to make inferences, predictions and conclusions

MG2H-8 solves problems involving counting techniques, multistage events and expectation

MG2H-9 chooses and uses appropriate technology to locate and organise information from a range of contexts

MG2H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

PB2 Multistage events and applications of probability

Terminology

arrangement	financial loss	simulation
expected number	multiplication principle	tree diagram
expected value	multistage event	trial
experimental result	ordered selection	two-stage event
financial expectation	probability tree diagram	unordered selection
financial gain	sample space	

Use of technology

Students should use probability simulations for large numbers of trials, eg tossing a coin, or rolling a die, two hundred times. The graphing facility of a spreadsheet could be used to investigate results.

Students could use appropriate technology to develop simulations of experiments such as those listed above.

Notes

Formulae involving factorial notation are not required in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.

Students will have different levels of familiarity with probability contexts such as those involving dice and card games.

While games of chance are to be investigated, the small chance of winning large prizes in popular lotteries and number draws should be emphasised.

'Probability tree diagrams' can be developed as a shorthand for tree diagrams in which every branch represents an equally likely event. Using the multiplication principle and a tree diagram in which every branch represents an equally likely event establishes the conceptual background for multiplying along branches of a probability tree diagram.

PB2 Multistage events and applications of probability

The focus of this topic is on counting the number of outcomes for an experiment, or the number of ways in which an event can occur, and the calculation of outcomes expected from simple experiments and comparing them with experimental results. The probability of particular outcomes can then be established.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-8, MG2H-9, MG2H-10

Content

Students:

- multiply the number of choices at each stage to determine the number of outcomes for a multistage event
- establish that the number of ways in which n different items can be arranged in a line is $n(n-1)(n-2)\ldots\times 1$, eg the number of arrangements of four different items is $4\times 3\times 2\times 1=24$; the number of arrangements of three different items is $3\times 2\times 1=6$
- construct and use tree diagrams to establish the outcomes for a simple multistage event
- establish the number of ordered selections that can be made from a group of different items (small numbers only), eg if selecting two particular positions (such as captain and vice-captain) from a team of five people, the number of selections is $5\times 4=20$
- establish the number of unordered selections that can be made from a group of different items (small numbers only), eg if selecting a pair of people to represent a team of five, the number of selections is half of the number of ordered selections
- use the formula for the probability of an event to calculate the probability that a particular selection will occur
- use probability tree diagrams to solve problems involving two-stage events
- calculate the expected number of times a particular event would occur, given the number of trials of a simple experiment, by establishing the theoretical probability of the event and multiplying by the number of trials
- compare the result in the previous dot point with an experimental result
- calculate expected value by multiplying each outcome by its probability and adding the results together.

Considerations

- Class discussion should include whether a particular event is obviously dependent or independent, eg a set of free throws in basketball. Some people are of the view that the success of each shot is independent of the result of the last shot. Others suggest that there is a dependent psychological impact of success or failure based on the result of the last shot. Despite many statistical studies of basketball free throws, neither view has been established as correct. There may be similar implications for the number of faults served in a tennis match.
- Establishing the number of arrangements (ordered and unordered) that can be obtained should include the use of tree diagrams and making lists.
- Expected value includes financial expectation calculations.
- A financial loss is regarded as a negative.
- When playing games of chance, any entry fee into a game is considered a financial loss.

Suggested applications

- In how many ways can the names of three candidates be listed on a ballot paper? What is the probability that a particular candidate's name will be at the top of the paper? Check by listing.
- Forty people are given Drug X for the treatment of a disease. Drug X has a success rate of 80%. How many patients can be expected to be treated successfully?

  Solution

  Number of patients expected to be treated successfully $= 40 \times 0.80 = 32$.
- Data collected in a certain town suggests that the probabilities of there being 0, $1, 2, 3, 4$ or 5 car thefts in one day are $0.10, 0.35, 0.30, 0.08, 0.15$ and 0.02, respectively. What is the expected number of car thefts occurring on any particular day?

  Solution

  Expected number of car thefts

  $$= 0 \times 0.10 + 1 \times 0.35 + 2 \times 0.30 + 3 \times 0.08 + 4 \times 0.15 + 5 \times 0.02$$
  $$= 1.89$$

  The expected number of car thefts is 1.89 cars per day.
- Given the following options:

  (A) a sure gain of $\$300$

  (B) a 30% chance of gaining $\$1000$ and a 70% chance of gaining nothing,

  calculate the financial expectation of option A and of option B. In this example, the financial expectations are equal. Given a choice, many people would select option A because it is certain. This example illustrates that many decisions are not made based on financial expectation alone.
- Paul plays a game involving the tossing of two coins. He gains $\$5$ if they both show heads and $\$1$ if they show a head and a tail, but loses $\$6$ if they both show tails. What is his financial expectation for the game?

Strand: Algebra and Modelling

Algebra is fundamental to generalisation in mathematics and to the solution of problems across subject areas. It is an essential tool in problem-solving through the solution of equations, the graphing of relationships, and modelling with functions. In finance and business, for example, 'break-even' analysis relies on algebraic representation.

In the Algebra and Modelling Strand in the HSC Mathematics General 2 course, students develop and apply algebraic skills and techniques to interpret and use linear and non-linear mathematical models in a range of vocational and other practical contexts.

Outcomes addressed

A student:

MG2H-3	makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions
MG2H-9	chooses and uses appropriate technology to locate and organise information from a range of contexts
MG2H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

AM3	Further algebraic skills and techniques
AM4	Modelling linear relationships
AM5	Modelling non-linear relationships

Terminology

algebraic fraction
break-even
denominator
direct variation
elimination method
exponential function
exponential growth
extrapolate
gradient
index
index law
interpolate
inverse variation
like term
linear equation
maximum
minimum
quadratic function
simultaneous equations
substitution method
y-intercept

Use of technology

Students should have access to appropriate technology in order to create graphs of functions, including linear, quadratic, cubic, hyperbolic, and exponential functions. They can change the coefficients and constant in the algebraic representation of a function to observe and explain the effect on the graph of the function.

Students should identify ordered pairs on a graph by 'tracing'. The 'zoom' feature in geometry software or on a graphing calculator could be used to explore graphs and to find points of intersection, eg in break-even analysis.

Notes

Algebraic skills should be developed through vocational and other practical contexts.

The concept of variation is widely used in many applications of algebra in further studies and in the workplace. As a concept, it could be covered within AM5 *Modelling non-linear relationships.*

The Focus Studies in this course provide a range of opportunities for students to apply knowledge, skills and understanding developed in the Algebra and Modelling Strand.

AM3 Further algebraic skills and techniques

In this topic, students develop algebraic skills and techniques that have applications in work-related and everyday contexts. These skills and techniques include application of the index laws, solution of linear equations, and more difficult examples of substitution into vocational formulae.

Outcomes addressed

MG2H-3, MG2H-9, MG2H-10

Content

Students:

- add and subtract like terms, including like terms involving powers, eg $3x^2 + 2y - 15x^2 - 3y$
- add and subtract simple algebraic fractions with different numerical denominators,

  eg $\frac{x}{6} + \frac{x}{4}$, $\frac{7x}{10} - \frac{x}{3}$
- establish and apply index laws in algebraic form

  $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $\left(a^m\right)^n = a^{mn}$
- apply index laws to simplify expressions, eg $2x^0 + 4$, $4b^4 \times \frac{1}{4}b^3$, $\frac{3x^4y}{6x^2y^3}$
- expand and simplify algebraic expressions, eg $4x^2\left(3x^3 - 2\right) - 3x^2\left(x^3 + 9\right)$
- solve linear equations involving up to four steps, including with unknowns in the denominator, eg $\frac{x}{3} + 3 = 2x - 9$, $\frac{w}{2} + \frac{w}{5} = -7$, $\tan 30° = \frac{45}{h}$
- solve equations following substitution of values,

  eg find the value of R_1 if $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, $R = 1.12$ and $R_2 = 2.24$
- change the subject of a formula,

  eg make r the subject of $v = \sqrt{\frac{mr}{\pi}}$
- solve simple linear simultaneous equations.

Considerations

- Formulae should be drawn from vocational and other practical contexts, eg

$$V = IR - E \qquad Z = \sqrt{R_2 + (2\pi f L)^2}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}} \qquad e = iR + \frac{Q}{C}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$$

- Both the substitution and elimination methods should be used to solve pairs of simple simultaneous equations, eg

$$\begin{Bmatrix} y = 2x \\ 3x + 2y = 7 \end{Bmatrix}, \begin{Bmatrix} y = 2x - 5 \\ y = 3x + 2 \end{Bmatrix}, \begin{Bmatrix} 3x + y = 13 \\ x - y = 3 \end{Bmatrix}, \begin{Bmatrix} 2x + 3y = 14 \\ 2x - y = 6 \end{Bmatrix}$$

Suggested applications

- Students construct graphs of simultaneous linear equations (eg graphs of costs and income) to solve problems involving the identification of break-even points, eg finding the time period following installation for an appliance to start saving money for a household.
- Students change the subject of formulae and evaluate by substitution, eg
  - make F the subject of the formula $C = \frac{5}{9}(F - 32)$, and evaluate F given that $C = 100$
  - make C the subject of the formula $X = \frac{1}{2\pi f C}$, and evaluate C to three significant figures given that $X = 0.02$ and $f = 1.85$.

AM4 Modelling linear relationships

This topic focuses on the solution of practical problems arising from situations that can be modelled algebraically and graphically using linear functions. Students develop knowledge, skills and understanding in the graphing of functions in different contexts.

Outcomes addressed

MG2H-3, MG2H-9, MG2H-10

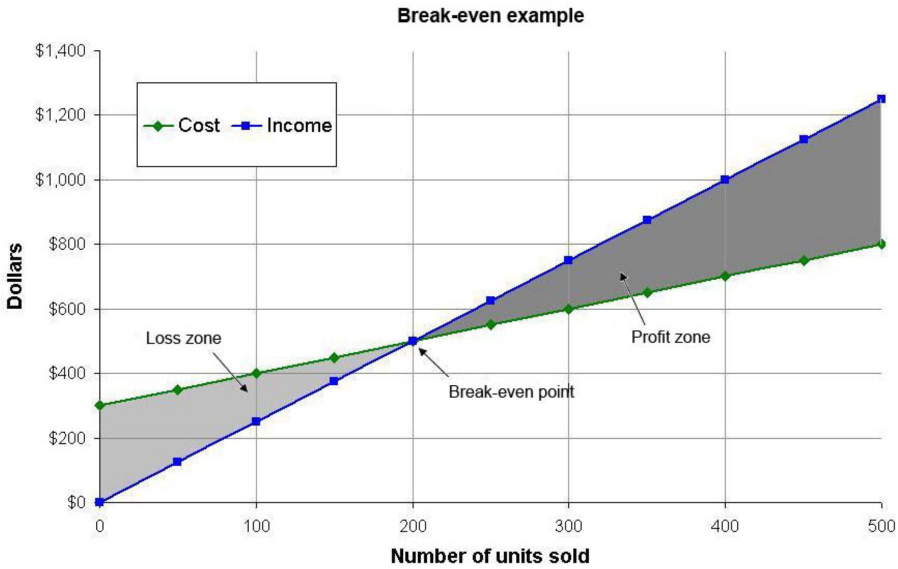
Content

Students:

- generate tables of values for linear functions (including for negative values of x)
- graph linear functions for all values of x with pencil and paper, and with graphing software
- develop graphs of linear equations of the form $y = mx$ from descriptions of situations in which one quantity varies directly with another
- use the graph in the previous dot point to establish the value of m (the gradient) and to solve problems related to the given variation context
- interpret linear functions as models of physical phenomena
- establish the meaning of the gradient and the y-intercept for a given practical context
- develop linear equations from descriptions of situations in which one quantity varies directly with another
- solve contextual problems involving linear models
- interpret the point of intersection of the graphs of two linear functions drawn from practical contexts
- solve contextual problems using a pair of simple linear simultaneous equations
- develop and use linear functions to model physical phenomena
- recognise the limitations of models when interpolating and/or extrapolating
- apply break-even analysis to simple business problems that can be modelled with linear and quadratic functions.

Considerations

- In the teaching of break-even analysis, students should learn that *profit = income function – cost function*. They should be able to recognise and interpret the income equation and cost equation drawn on the same graph. Students solve break-even problems graphically, with questions emphasising the break-even point, the profit zone, the loss zone, and interpretation of the gradients, x-intercept and y-intercept.



An algebraic treatment of break-even analysis is also required. The income function is a simple linear function of the form $I = mx$, where x is the number of units sold and m is the selling price per unit sold. The cost function is of the form $C = mx + b$, where x is the number of units sold, m is the cost price per unit manufactured, and b is the fixed costs of production. The point of intersection of I and C is the break-even point. Consider the scenario depicted in the graph above. Martha sells muffins for $\$2.50$ each. It costs $\$1.25$ to make each muffin and $\$300$ for the equipment needed to make the muffins.

In this example, the income function is $I = 2.5x$ and the cost function is $C = 1.25x + 300$. The break-even point is the solution of the equation $I = C$, which can be solved graphically or algebraically. The profit or loss can be calculated using *profit (loss) = income – costs.*

Suggested applications

- Use distance/velocity/time graphs to solve a range of problems.
- Students investigate the question: 'Does the approximation method "double and add $30°$" for converting from degrees Celsius to degrees Fahrenheit always give an answer close to the correct answer?' The formula for converting degrees Celsius to degrees Fahrenheit could be graphed, along with the formula arising from the 'rule of thumb'.

  This question could also be investigated using a spreadsheet and/or a graphing calculator.

AM5 Modelling non-linear relationships

This topic focuses on the solution of practical problems arising from situations that can be modelled algebraically and graphically using non-linear functions. Students develop knowledge, skills and understanding in the graphing of functions in different contexts.

Outcomes addressed

MG2H-3, MG2H-9, MG2H-10

Content

Students:

- use a graph of a quadratic function to find maximum and minimum values in practical contexts
- recognise, graph and compare, by completing tables of values, the properties of the graphs of:

  $y = ax^2 + c$, for all values of x

  $y = ax^3 + c$, for all values of x

  $y = \frac{a}{x}, a > 0$, for all values of x

  $y = b\left(a^x\right), \ x \geq 0$

- solve contextual problems involving exponential growth
- use algebraic functions (as described above) to model physical phenomena
- recognise the limitations of models when interpolating and/or extrapolating
- develop equations of the form $y = ax^2$, $h = at^3$ from descriptions of situations in which one quantity varies directly as a power of another
- develop equations such as $y = \frac{a}{x}$ from descriptions of situations in which one quantity varies inversely with another
- evaluate a in the equations shown in the previous two dot points, given one pair of variables, and use the resulting formula to find other values of the variables.

Considerations

- Graphing software should be used to investigate various quadratic functions and their maximum and minimum values.
- Graphing software can be used to vary coefficients and constants of the various functions addressed in this topic to observe changes to the graphs of the functions.
- Variation problems should be presented in a number of formats, including in written, tabular and graphical form.
- When modelling physical phenomena, the independent variable will usually be limited to positive values and zero.

Suggested applications

- Sketch at least 10 rectangles that have the same perimeter. Record length versus area in a table. Sketch the resulting function and use the graph to determine the rectangle with maximum area. Describe this rectangle.
- Investigate models of population growth.
- Sketch a function showing the growth of a population of bacteria.
- Investigate compound interest as the time period shortens.
- On the Earth, the equation $d = 4.9t^2$ can be used to express the distance (d metres) that an object falls in t seconds, if air resistance is ignored. Investigate the equations for the moon and for other planets: eg on the moon, the equation is $d = 0.8t^2$. Create a table of values for the function $d = 4.9t^2$ either manually or by using a spreadsheet, and use the table to answer questions such as: How long does it take for an object to fall 300 m?
- Solve problems where two quantities vary in a related way, eg the number of eggs used in a recipe for a particular cake varies with the square of the diameter of the tin, for tins with constant depth. If two eggs are used in a recipe for a tin of diameter 15 cm, how many eggs would be used for a tin of diameter 35 cm?
- Cubic models can be used to estimate the mass of objects. For example, could you lift a cubic metre of cork? An expression for the mass in grams, M, of a cube of cork is $M = 0.25x^3$, where x is the side length of the cube in centimetres.
- Inverse variation can be used to find how much each person contributes when a cost is shared. For example, a household has $\$306$ in bills. Create a table and draw a graph to show how much each person pays if there are 2, 3, 4 or 5 people contributing equally to pay the bills.
- An exponential expression such as $M = 1.5(1.2)^x$ can be used to calculate the mass M kg of a baby orangutan at age x months. This model applies for a limited time, up to $x = 6$. Calculate the mass of a baby orangutan at age three months.

Focus Study: Mathematics and Health

Mathematical knowledge and skills are essential in the study of medicine and the human body, and in medical practice. This includes the accurate administering of appropriate amounts of medicines based on such calculations as body mass, and the ability to read and interpret medical charts and tables. Knowledge and understanding of matters such as life expectancy are important in developing an appropriate understanding of insurance and its associated costs.

In the Mathematics and Health Focus Study, students apply, and develop further, knowledge, skills and understanding in Data and Statistics, Measurement, Probability, and Algebra and Modelling to contexts related to health.

Outcomes addressed

A student:

MG2H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts
MG2H-2	analyses representations of data in order to make inferences, predictions and conclusions
MG2H-3	makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions
MG2H-5	interprets the results of measurements and calculations and makes judgements about reasonableness, including the degree of accuracy of measurements and calculations and the conversion to appropriate units
MG2H-7	answers questions requiring statistical processes, including the use of the normal distribution, and the correlation of bivariate data
MG2H-9	chooses and uses appropriate technology to locate and organise information from a range of contexts
MG2H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

FSHe1	Body measurements
FSHe2	Medication
FSHe3	Life expectancy

Terminology

bias
biometric data
bivariate data
causality
concentration
correlation
correlation coefficient
dosage
dosage strength
drip rate
extrapolation
interpolation
least-squares line of best fit
life expectancy
line of fit
linear
mass
medication
ordered pair
outlier
ratio
regression line
scatterplot
standard deviation
strength of association
trendline
y-intercept

Use of technology

Appropriate technology should be used to construct, and determine the equation of, a line of fit and least-squares line of best fit, and to calculate correlation coefficients.

Teachers should demonstrate the 'trendline' function on a spreadsheet and then have students explore the function with their own sets of data.

Australian Institute of Health and Welfare data can be accessed on the internet for use in FSHe3 *Life expectancy.*

Notes

Learning and teaching, and assessment, should draw on and integrate the mathematics in Strands within the course, eg Data and Statistics, Measurement, and Algebra and Modelling.

Materials used for learning and teaching, and assessment, should include current information from a range of sources, including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

This Focus Study provides students with special interest in the health sciences, PDHPE or Visual Arts the opportunity to explore aspects of mathematics involved in those areas.

FSHe1 Body measurements

In this topic, students construct scatterplots and lines of fit and use them to explore relationships and make predictions. They use technology to calculate correlation coefficients and interpret the strength of association of variables, and to calculate the equation for the line of best fit using the least-squares method.

Outcomes addressed

MG2H-2, MG2H-3, MG2H-5, MG2H-7, MG2H-9

Content

Students:

- investigate biometric data obtained by measuring the body and by accessing published data
- plot ordered pairs of body measurement data onto a scatterplot by hand and by using appropriate technology
- recognise patterns in a scatterplot of body measurements, eg
  - whether the points appear to form a mathematical pattern
  - whether the pattern appears to be linear
- estimate and draw 'by eye' a line of fit on a scatterplot
- calculate correlation coefficients for different body measurements using appropriate technology (students are not required to calculate correlation coefficients by hand)
- interpret the strength of association for different body measurements using a given correlation coefficient
- interpret the sign of a given correlation coefficient
- construct the least-squares line of best fit
- determine the least-squares line of best fit using the correlation coefficient (r), the mean of the x scores, and the mean of the y scores, and the standard deviation of the x scores and the standard deviation of the y scores

  least-squares line of best fit $y = gradient \times x + y\text{-}intercept$ where:

  r is the correlation coefficient, calculated using appropriate technology

  $gradient = r \times \dfrac{standard\ deviation\ of\ y\ scores}{standard\ deviation\ of\ x\ scores}$ and the

  $y\text{-}intercept = \bar{y} - (gradient \times \bar{x})$

- use a least-squares line of best fit to interpolate
- recognise that a high degree of correlation for different body measurements does not necessarily imply causality.

Considerations

- The biometric data obtained should be used to construct lines of fit by hand. The work in relation to lines of fit is extended to include the least-squares line of best fit and the determination of its equation using appropriate technology.
- Outliers need to be examined carefully, but should not be removed unless there is a strong reason to believe that they do not belong in the data set.
- The least-squares line of best fit is also called the regression line. This is the line that lies closer to the data points than any other possible line (according to a standard measure of closeness).
- It should be noted that the predictions made using a line of best fit:
  - are more accurate when the correlation is stronger and there are many data points
  - should not be used to make predictions beyond the bounds of the data points to which it was fitted
  - should not be used to make predictions about a population that is different from the population from which the sample was drawn.
- The trendline feature of a spreadsheet graph should be explored, including the display of the trendline equation (which uses the least-squares method). This feature also allows lines of fit that are non-linear.
- The forecast function on a spreadsheet can be used to make predictions in relation to bivariate data.
- It should be noted that correlation is not limited to body measurements. Discussion of correlation beyond body measurements may be necessary to illustrate positive and negative relationships. Correlation is further explored within the Focus Study in FSHe3 *Life expectancy*.

Suggested applications

- Students could measure body dimensions such as arm-span, height and hip-height, as well as length of stride. It is recommended that students have access to published biometric data to provide suitable and realistic learning contexts. Comparisons could be made using parameters such as age or gender.
- Biometric data could be extended to include the results of sporting events, eg the progression of world-record times for the men's 100-metre freestyle swimming event.
- Predictions could be made using the line of fit, eg predicting a person's height based on their arm-span or arm-length. Students should assess the accuracy of the predictions by measurement and calculation in relation to an individual not in the original data set, and by the value of the correlation coefficient.

FSHe2 Medication

In this topic, students perform a range of calculations related to child and adult medication. They apply various formulae in the solution of practical problems.

Outcomes addressed

MG2H-2, MG2H-5

Content

Students:

- recognise the need for units of mass smaller than the gram
- convert grams (g) to milligrams (mg), and vice versa
- perform calculations involving concentrations expressed as mass/volume, eg 100 mg/5 mL or 100 mg in 5 mL
- calculate required dosages for children and adults from packets given age or weight
- calculate required dosages for children using various formulae
- calculate drip rates and dosage strengths.

Considerations

- Examples of dosage panels from over-the-counter medications should be examined.
- Students should develop a clear understanding of formulae used to calculate required dosages for children and the variables included in the formulae. Such formulae include Fried's formula, Young's formula and Clark's formula.
  - *Children 1–2 years (Fried's formula):*

    $$Dosage\ for\ children\ 1\text{–}2\ years = \frac{age\ (in\ months) \times adult\ dosage}{150}$$

  - *Children 1–12 years (Young's formula):*

    $$Dosage\ for\ children\ 1\text{–}12\ years = \frac{age\ of\ child\ (in\ years) \times adult\ dosage}{age\ of\ child\ (in\ years) + 12}$$

  - *General formula (Clark's formula):*

    $$Dosage = \frac{weight\ in\ kg \times adult\ dosage}{70}$$

- Drip rate questions should include solving for different variables, eg How long will the drip need to run? At what rate will the drip flow? How much fluid will be given?

Suggested applications

- Students make calculations from dosage panels, including the amount per dose and the frequency of dosage.
- Students calculate dosages for different medication types, eg oral medication in liquid or tablet form.
  - *Example for medication in liquid form*

    A patient is prescribed 1000 mg of a mild painkiller. The medication available contains 100 mg in 5 mL. How much medication should be given to the patient?

    1. Using a formula

    Solution: $volume\ required = \frac{strength\ required}{stock\ strength} \times \frac{volume\ of\ stock}{1} = \frac{1000}{100} \times \frac{5}{1} = 50$ mL

    2. Using a ratio approach

    Solution: 5 mL: 100 mg = $x : 1000$ mg, $x = 50$ mL

  - *Example for medication in tablet form*

    A patient is prescribed 750 mg of a medication. Tablets, each of 500 mg, are available. How many tablets should be given?

    Solution: $volume\ required = \frac{strength\ required}{stock\ strength} = \frac{750}{500} = 1.5$ tablets

  - *Example for medication in flow rate form*

    A patient is to receive 1.5 litres of fluid over 8 hours. What is the required flow rate in mL/h?

    Solution: $required\ flow\ rate = \frac{volume\,(mL)}{time\,(h)} = \frac{1500}{8} = 188$ mL/h

- Students could calculate drip rates and dosage strengths using nursing formulae.

FSHe3 Life expectancy

Life expectancy is the number of years a person of a particular age today can expect to live, on average.

In this topic, students perform a range of calculations related to life expectancy. They apply various mathematical techniques, including modelling, to interpret life expectancy data and make relevant predictions.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-3, MG2H-7, MG2H-9, MG2H-10

Content

Students:

- interpret life expectancy data in various forms, including in tables and graphs
- plot life expectancy data for a range of variables (eg country of birth and country's population, gross domestic product (GDP), birth rate, infant mortality rate, spending on health care, percentage of urbanisation, etc), using the most appropriate form of display
- investigate trends, or points of significance, for specific countries over time, including any specific historical events such as medical advancements
- interpolate from plotted data to make predictions where appropriate
- interpret published graphs and statistics, including critically evaluating data collection methods, eg bias in data that may be included or omitted
- construct scatterplots of life expectancy for a range of variables, eg year of birth, gender, health status, etc
- create scatterplots for sets of variables to identify strong predictors of life expectancy, and calculate correlation coefficients
- plot life expectancy for different variables using online life expectancy calculators to make assessments about how variables are weighted, eg What effect does smoking have on a person's life expectancy?

Considerations

- Students should begin this topic by developing an understanding of what is meant by life expectancy. They should be encouraged to make predictions about the variables that they think may affect life expectancy, both on a global and on a personal level. Teachers may wish to brainstorm a list for investigation during work on the Focus Study.
- Life expectancy is calculated using population census data. The historical basis of life expectancy records should be discussed. In 1662, John Graunt published *Natural and Political Observations Made upon the Bills of Mortality.* The work was based on the publication of the number of people dying each week in big cities, with the data categorised in terms of the cause of death. Discussion should include the validation of Graunt's data-gathering method and Graunt's method in comparison to current data-collection methods.
- Life expectancy data can be displayed in various forms, eg colour-coded maps of the world indicating life expectancy for individual countries, scatterplots of life expectancy against year of birth or country of birth, and line graphs of life expectancy against infant birth rate for various countries.
- Historical events may include medical advancements (eg the development of vaccines), periods of conflict, and technological advancements. Graphs of Australian male life expectancy since 1891 reflect an increase in deaths from road accidents in the 1950s and 1960s.
- It should be noted that not all relationships regarding life expectancy will be linear, eg graphs of gross domestic product (GDP) against life expectancy are non-linear.

Suggested applications

- Students use online life expectancy calculators to identify variables associated with life expectancy.
- Students use spreadsheets to create graphs displaying life expectancy data.
- Analysts have made predictions that life expectancy in some Western countries may decline over the next few decades with increases in the incidence of weight-related diseases. Students could model the effect that such increases may have on future life expectancies.
- Students explore providing a mathematical justification for the insurance premiums of women being lower than those of men because, in general, they live longer than men.

Focus Study: Mathematics and Resources

The efficient use of energy in the home and the workplace is a major and increasing area of concern for individuals and communities. Householders can reduce their energy bills and contribute to the sustainability of resources by employing strategies designed to reduce waste and promote recycling. Mathematical applications are used in relation to many aspects of resource usage, including in the estimation of areas and volumes in the context of water supply.

In the Mathematics and Resources Focus Study, students apply, and develop further, knowledge, skills and understanding in Financial Mathematics, Data and Statistics, Measurement, and Algebra and Modelling to contexts involving resources.

Outcomes addressed

A student:

MG2H-1 uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts

MG2H-2 analyses representations of data in order to make inferences, predictions and conclusions

MG2H-3 makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions

MG2H-4 analyses two-dimensional and three-dimensional models to solve practical problems, including those involving spheres and non-right-angled triangles

MG2H-5 interprets the results of measurements and calculations and makes judgements about reasonableness, including the degree of accuracy of measurements and calculations and the conversion to appropriate units

MG2H-7 answers questions requiring statistical processes, including the use of the normal distribution, and the correlation of bivariate data

MG2H-9 chooses and uses appropriate technology to locate and organise information from a range of contexts

MG2H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Content summary

FSRe1 Water availability and usage

FSRe2 Dams, land and catchment areas

FSRe3 Energy and sustainability

Terminology

aerial photograph
appliance
availability
Building Sustainability Index (BASIX)
catchment
consumption
energy efficient
energy rating
grid-square method
kilolitre
kilowatt
kilowatt-hour
polygon method
power rating
running cost
Simpson's rule
sustainability
usage
watt
watt-hour

Use of technology

Students should use spreadsheets to create graphs to display two or more sets of data, eg the rainfall in Cape York compared to the rainfall in Broken Hill.

Students could use spreadsheets or other appropriate software to calculate summary statistics.

The internet can be used to obtain suitable data for use in the study of the topics within this Focus Study.

Google Maps is an online resource that can be used to access maps and scaled aerial photographs, and to calculate distances on land.

Notes

Water is an essential resource for life. In this Focus Study, students study aspects of water as a resource, including rainfall, personal water usage, and local, state and national water usage, and compare rainfall and water usage in a range of countries.

Electricity is a fundamental resource in modern life. In this Focus Study, students solve problems based on the consumption of household electricity.

Current information on the Building Sustainability Index (BASIX) Certificate is available on the internet.

FSRe1 Water availability and usage

In this topic, students interpret information, make comparisons, and perform a range of calculations in relation to requirements and data relevant to water availability and usage.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-3, MG2H-4, MG2H-5, MG2H-7, MG2H-9, MG2H-10

Content

Students:

- interpret information about a household's water usage, eg a household water bill
- collect and interpret data and calculate statistics on household and personal water usage
- investigate household water usage in different Australian and international locations
- construct and interpret rainfall graphs
- calculate the probability of rainfall in a locality
- compare rainfall in different regions, states and countries
- collect and interpret data and calculate statistics on water availability and usage at local, state and national levels
- calculate the volume of water held by tanks of various shapes and sizes
- investigate the costs of water usage at local, state and national levels, using published data.

Considerations

- Rainfall data is widely available on the internet. Useful information and data can be found at the Australian Bureau of Meteorology and Sydney Water websites.

Suggested applications

- Students could use rainfall data to create graphs or to calculate summary statistics using knowledge, skills and understanding from the Data and Statistics Strand – eg rainfall data over three years.
- Collect and tabulate rainfall data to predict the probability of rain, eg by tabulating rainfall data from the Bureau of Meteorology, the number of days with rain in Mudgee for the past three summers can be found. This data can be used to predict the probability of rain in summer in Mudgee. This could easily be extended to predict the probability of two rainy days in succession in summer in Mudgee.
- Determine the amount and cost of water used by various household activities, including showering and bathing, washing clothes, watering the garden, washing a car, and using the toilet.
- Students log their water use over a one-week period and use this to estimate personal water usage and costs over longer time periods.
- Students calculate the volume of water that can be collected on a roof given the rainfall in millimetres and the roof area (plan view area).

FSRe2 Dams, land and catchment areas

In this topic, students perform a range of calculations involving scale, length, area and volume in relation to land and catchment areas and water storage. They learn to apply estimation methods in solving various problems involving area and volume.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-3, MG2H-4, MG2H-5, MG2H-10

Content

Students:

- calculate the scale used on a photograph given that it contains features with standard dimensions, eg an Olympic swimming pool
- calculate the perimeter of a section of land using a site plan or aerial photograph that includes a scale
- calculate the distance between two points on a section of land using online tools
- estimate the area of land and catchment areas
- calculate actual areas using scale diagrams
- calculate the volume of rainfall using $V = Ah$
- estimate the volume of a reservoir or dam using Simpson's rule (up to two applications)

$$V = \frac{h}{3} \times \left\{ Area_{LEFT\ END} + 4 \times Area_{MIDDLE} + Area_{RIGHT\ END} \right\}.$$

Considerations

- Students can access suitable websites to obtain appropriate maps and photographs.
- Online tools such as those on the Google Maps website can be used to determine distances on the surface of the Earth.
- The following estimation methods can be used to calculate the approximate area of a piece of land, a catchment (drainage basin), or a dam:
  (i) the grid-square method
  (ii) the polygon method.
- Sources of error in determining areas need to be considered, eg errors in the measurement of distances.
- In addition to using Simpson's rule to calculate volumes, students can approximate the volume of a dam by using the product of the surface area of the dam and its average depth.

Suggested applications

- Use a map or aerial photograph where a feature has known dimensions (eg an Olympic swimming pool, an athletics field, or a playing field for a particular sport) to find the scale of the map or photograph.

FSRe3 Energy and sustainability

The principal focus of this topic is the calculation and comparison of household electricity consumption and costs, and the calculation and interpretation of related statistics.

Outcomes addressed

MG2H-1, MG2H-2, MG2H-5, MG2H-9, MG2H-10

Content

Students:

- interpret information about a household's electricity usage, eg a household electricity bill
- rank common appliances and physical activities in terms of their energy consumption in watts
- describe the watt-hour and kilowatt-hour as units of energy usage
- calculate the cost of running different household appliances for various time periods, given the power rating, usage time, and cost of power
- perform calculations and conversions involving units related to power, eg watt, watt-hour
- interpret the energy rating of appliances and compare running costs of different models of the same type of appliance
- calculate and interpret summary statistics for electricity costs, production data and consumption data at local, state, national and international levels
- investigate local council requirements for energy-efficient housing
- calculate building sustainability measures based on the requirements of the Building Sustainability Index (BASIX) Certificate
- identify the issues addressed in the BASIX, eg area of site, water, thermal comfort and energy.

Considerations

- The watt is the International System of Units (SI) derived unit of power and is equal to one joule per second. By definition, power is a rate. The symbol for the watt is W.
- The energy consumptions of common appliances and physical activities are to be ranked by students.

  Examples that could be used include:
  - a typical household incandescent light bulb uses 40 to 100 watts
  - a person climbing a flight of stairs is doing work at the rate of approximately 200 watts
  - a highly-trained athlete can work at up to approximately 2000 watts for brief periods
  - a car engine produces approximately 25 000 watts while cruising.
- One watt-hour is the amount of energy (usually electrical) expended by a one-watt load (eg a light bulb) drawing power for one hour. The watt-hour (symbol W·h or Wh) is a unit of energy. It is most commonly used on household electricity meters in the form of the kilowatt-hour (kW·h or kWh), which is 1000 watt-hours.
- Quantities and units may be expressed in both decimal form and scientific notation, eg 6.8×10^3 MW or 6 800 000 kW.
- Building plan dimensions are expressed in millimetres.
- Conversion of units is to be based on the table below:

Table of units		
Multiple	**Name**	**Symbol**
10^{-3}	milliwatt	mW
10^{0}	watt	W
10^{3}	kilowatt	kW
10^{6}	megawatt	MW
10^{9}	gigawatt	GW

Suggested applications

- Calculations of running costs of various appliances, eg
  1. Calculate the cost of running a 200-watt television set for six hours if the average peak rate for domestic electricity is \$0.15/kWh.

     Solution

     Total electricity used = 200 x 6 = 1200 watt-hours or 1.2 kWh.

     Cost of electricity used is 1.2 kWh × \$0.15/kWh = \$0.18.
  2. Calculate the cost of running a 2400-watt (2.4 kW) fan heater for eight hours per day for 30 days. Assume electricity is charged at \$0.18/kWh.

     Solution

     Total electricity used = 2.4 x 8 x 30 = 576 kWh.

     Cost of electricity used is 576 kWh x \$0.18/kWh = \$103.68.
- Data in relation to worldwide electricity consumption is available on the internet. Students could be asked to extract data from tables or graphs and interpret this information in order to compare electricity consumption in different countries.

11 HSC Mathematics General 2: Assessment and Reporting

Advice on appropriate assessment practice in relation to the HSC Mathematics General 2 course (and the Preliminary Mathematics General course) is contained in the document *Assessment and Reporting in the HSC Mathematics General 2 Course.*

This document provides general advice on assessment in Stage 6, as well as the specific requirements for the HSC Mathematics General 2 course (and the Preliminary Mathematics General course). The document contains:

- suggested components and weightings for the internal assessment of the Preliminary Mathematics General course
- mandatory components and weightings for the internal assessment of the HSC Mathematics General 2 course
- the HSC examination specifications for the Preliminary Mathematics General/HSC Mathematics General 2 pathway, which describe the format of the external HSC examination.

This document and other resources and advice related to assessment for the Preliminary Mathematics General/HSC Mathematics General 2 pathway are available on the Board's website at www.boardofstudies.nsw.edu.au/syllabus_hsc.

12 Preliminary Mathematics General/ HSC Mathematics General 1 Pathway

12.1 Pathway Structure

The following overview illustrates the structure of the Preliminary Mathematics General/HSC Mathematics General 1 pathway.

Preliminary Mathematics General Course*		HSC Mathematics General 1 Course (CEC)	
Strand:	**Financial Mathematics**	**Strand:**	**Financial Mathematics**
FM1	Earning and managing money	FM4CEC	Credit cards
FM2	Investing money		
FM3	Taxation		
Strand:	**Data and Statistics**	**Strand:**	**Data and Statistics**
DS1	Statistics and society, data collection and sampling	DS4CEC	Distributions
DS2	Displaying and interpreting single data sets	DS5CEC	Interpreting sets of data
DS3	Summary statistics	DS6CEC	Working with statistics
Strand:	**Measurement**	**Strand:**	**Measurement**
MM1	Units of measurement and applications	MM4CEC	Further applications of area and volume
MM2	Applications of perimeter, area and volume		
MM3	Similarity of two-dimensional figures, right-angled triangles		
Strand:	**Probability**	**Strand:**	**Probability**
PB1	Relative frequency and probability	PB2CEC	Multistage events and applications of probability
Strand:	**Algebra and Modelling**	**Strand:**	**Algebra and Modelling**
AM1	Algebraic manipulation	AM3CEC	Further algebraic skills
AM2	Interpreting linear relationships	AM4CEC	Modelling with functions
Focus Study:	**Mathematics and Communication**	**Focus Study:**	**Mathematics and Design**
FSCo1	Mobile phone plans	FSDe1CEC	Scale drawings and house plans
FSCo2	Digital download and file storage	FSDe2CEC	Design
Focus Study:	**Mathematics and Driving**	**Focus Study:**	**Mathematics and Household Finance**
FSDr1	Costs of purchase and insurance	FSHo1CEC	Accommodation costs: buying and renting
FSDr2	Running costs and depreciation	FSHo2CEC	Costs of running a household, maintenance and repairs
FSDr3	Safety		
		Focus Study:	**Mathematics and the Human Body**
		FSHu1CEC	Blood
		FSHu2CEC	Body measurements
		FSHu3CEC	Lung capacity
		Focus Study:	**Mathematics and Personal Resource Usage**
		FSPe1CEC	Water usage and collection
		FSPe2CEC	Electricity
		FSPe3CEC	Sustainability and energy-efficient housing
* The Preliminary Mathematics General course is undertaken by all students intending to study either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.			

12.2 Employability Skills

The Employability Skills build on the Mayer Key Competencies (developed in 1992), which attempted to describe generic competencies for effective participation in work. The *Employability Skills for the Future* report (March 2002) indicated that business and industry required a broader range of skills than the Mayer Key Competencies Framework. It featured an Employability Skills Framework identifying eight Employability Skills: *communication, teamwork, problem-solving, initiative and enterprise, planning and organising, self-management, learning,* and *technology.*

The Preliminary Mathematics General/HSC Mathematics General 1 pathway provides a context within which to develop general competencies considered essential for the acquisition of effective, higher-order thinking skills necessary for further education, work and everyday life.

Employability skills are embedded in the Preliminary Mathematics General and HSC Mathematics General 1 courses to enhance student learning. The employability skills are developed through the methodologies of the courses and through classroom pedagogy and reflect core processes of mathematical inquiry undertaken by students.

At all levels of the courses, students are developing the employability skill *learning*. As they engage with the various topics in the courses and related applications and modelling tasks, the employability skills *planning and organisation, communication, self-management, teamwork* and *initiative and enterprise* are developed. Through relevant course content and the advice provided on the selection and use of appropriate technology, students can develop the employability skill *technology*. Finally, students' continual involvement with seeking solutions to problems, both large and small, contributes towards their development of the employability skill *problem-solving*.

12.3 Use of Technology

The appropriateness, viability and level of use of different types of technology in the learning and teaching of courses within the Mathematics Key Learning Area are decisions for students, teachers and schools. However, the use of technology is encouraged in the learning and teaching, and school-based assessment, where appropriate, of courses within the learning area.

The Preliminary Mathematics General and HSC Mathematics General 1 courses contain advice and suggestions in relation to the use of a range of technology in the 'Use of technology', 'Considerations' and 'Suggested applications' sections within the course content. The courses provide a range of opportunities for the use of calculators and computer software packages in learning and teaching. This includes opportunities to utilise the graphing functions and financial and statistical capabilities of calculators, spreadsheets, and dynamic geometry and statistics software packages.

12.4 Course Requirements

The Preliminary Mathematics General/HSC Mathematics General 1 pathway consists of a preliminary course, Preliminary Mathematics General, of 120 indicative hours and an HSC course, HSC Mathematics General 1, of 120 indicative hours.

The Preliminary Mathematics General course is constructed on the assumption that students have experienced all of the Stage 5.1 content of the *Mathematics Years 7–10 Syllabus* (2002). Completion of the Preliminary Mathematics General course is a prerequisite for the study of the HSC Mathematics General 1 course (and the HSC Mathematics General 2 course).

Students may not study the Preliminary Mathematics General course or the HSC Mathematics General 1 course (or the HSC Mathematics General 2 course) in conjunction with any other mathematics course in Stage 6.

12.5 Presentation of Content

The course content for the HSC Mathematics General 1 course (and for the HSC Mathematics General 2 course) is organised into Strands and Focus Studies, with each of the Strands – Financial Mathematics, Data and Statistics, Measurement, Probability, and Algebra and Modelling – divided into *topics* that lead into the Focus Studies.

The Focus Studies are designed to be programmed over a continuous time period as they provide students with the opportunity to apply, and develop further, the knowledge, skills and understanding initially developed in the Strands, as well as introducing some new mathematical content. It is intended that students develop, through the Focus Studies, the capacity to integrate their knowledge, skills and understanding across the Strands.

The Preliminary Mathematics General course is undertaken by all students intending to study either the HSC Mathematics General 2 course or the HSC Mathematics General 1 course. The Preliminary Mathematics General course includes two Focus Studies: Mathematics and Communication, and Mathematics and Driving.

There are four Focus Studies in the HSC Mathematics General 1 course: Mathematics and Design, Mathematics and Household Finance, Mathematics and the Human Body, and Mathematics and Personal Resource Usage. (There are two Focus Studies in the HSC Mathematics General 2 course: Mathematics and Health, and Mathematics and Resources.)

The course content for the Preliminary Mathematics General/HSC Mathematics General 1 pathway (and for the Preliminary Mathematics General/HSC Mathematics General 2 pathway) is presented in the following format:

1. Initial facing pages for a Strand or Focus Study

Left page	***Right page***
Name of Strand or Focus Study A summary of the purpose/content of the Strand or Focus Study ***Outcomes addressed*** A list of course outcomes addressed in the study of the topic(s) in the Strand or Focus Study ***Content summary*** A list of the topic(s) studied within the Strand or Focus Study	***Terminology*** A list of key words and/or phrases met in the Strand or Focus Study, some of which may be new to students ***Use of technology*** Advice about the nature and use of technology that is appropriate to the teaching and learning of the topic(s) in the Strand or Focus Study ***Notes*** Notes relevant to teaching particular aspects of the Strand or Focus Study

2. Subsequent facing pages for a Strand or Focus Study

Left page	***Right page***
Name of Topic A summary of the purpose/content of the topic ***Outcomes addressed*** A list of course outcomes addressed in the study of the topic ***Content*** *Students:* The mathematical content to be addressed in the topic	***Considerations*** A list of important considerations for teaching and learning the topic ***Suggested applications*** A list of examples indicating the types of applications used to introduce and illustrate the mathematical content of the topic

13 Preliminary Mathematics General Course Content (see Section 9 for content detail)

Hours shown are indicative only.

Strands 80 hours

Financial Mathematics

FM1 Earning and managing money
FM2 Investing money
FM3 Taxation

Data and Statistics

DS1 Statistics and society, data collection and sampling
DS2 Displaying and interpreting single data sets
DS3 Summary statistics

Measurement

MM1 Units of measurement and applications
MM2 Applications of perimeter, area and volume
MM3 Similarity of two-dimensional figures, right-angled triangles

Probability

PB1 Relative frequency and probability

Algebra and Modelling

AM1 Algebraic manipulation
AM2 Interpreting linear relationships

Focus Studies 40 hours

Mathematics and Communication

FSCo1 Mobile phone plans
FSCo2 Digital download and file storage

Mathematics and Driving

FSDr1 Costs of purchase and insurance
FSDr2 Running costs and depreciation
FSDr3 Safety

Total indicative hours 120 hours

14 HSC Mathematics General 1 Course Content

Hours shown are indicative only.

Strands 40 hours

Financial Mathematics

FM4CEC	Credit cards

Data and Statistics

DS4CEC	Distributions
DS5CEC	Interpreting sets of data
DS6CEC	Working with statistics

Measurement

MM4CEC	Further applications of area and volume

Probability

PB2CEC	Multistage events and applications of probability

Algebra and Modelling

AM3CEC	Further algebraic skills
AM4CEC	Modelling with functions

Focus Studies 80 hours

Mathematics and Design

FSDe1CEC	Scale drawings and house plans
FSDe2CEC	Design

Mathematics and Household Finance

FSHo1CEC	Accommodation costs: buying and renting
FSHo2CEC	Costs of running a household, maintenance, and repairs

Mathematics and the Human Body

FSHu1CEC	Blood
FSHu2CEC	Body measurements
FSHu3CEC	Lung capacity

Mathematics and Personal Resource Usage

FSPe1CEC	Water usage and collection
FSPe2CEC	Electricity
FSPe3CEC	Sustainability and energy-efficient housing

Total indicative hours 120 hours

Strand: Financial Mathematics

A sound understanding of credit and the responsible use of credit cards is important in developing students' ability to make informed financial decisions.

In the Financial Mathematics Strand in the HSC Mathematics General 1 course, students read and interpret credit card statements, and perform a range of related financial calculations.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-2	analyses representations of data in order to make predictions
MG1H-6	makes informed decisions about financial situations likely to be encountered post-school
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts

Content summary

FM4CEC	Credit cards

Terminology

account balance
annual fee
annual interest rate
billing cycle
credit
credit card
credit card statement
daily interest rate
debit
debit card
fee
interest-free period
interest payable
ledger of spending
minimum payment
percentage annual interest rate
simple interest

Use of technology

Students should create a spreadsheet to simulate a credit card statement, including the calculation of the interest payable and the minimum payment due.

The internet should be used as a source of up-to-date information, eg interest rates and terms and conditions for different credit cards.

Notes

Teachers and students should have access to actual financial information and products, eg examples of credit card statements should be used in learning and teaching.

Students will study additional content in relation to the Financial Mathematics Strand in the Mathematics and Household Finance Focus Study within this course.

FM4CEC Credit cards

This topic focuses on the use of credit cards as a method of payment for goods and services. Students learn to read and interpret credit card statements, and to calculate the interest payable, the account balance, and the fees payable.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-6, MG1H-9

Content

Students:

- interpret credit card statements and carry out related calculations
- create a ‘ledger of spending’ that includes the date and cost of purchases and the payment method
- identify the various fees and charges associated with credit card usage, including interest charges, annual card fees, and late payment fees, and how they are calculated
- express a percentage annual interest rate as a daily interest rate in percentage and decimal form
- calculate simple interest for one billing cycle
- compare differences between credit cards and debit cards.

Considerations

- Students should have access to credit card statements issued by major Australian financial institutions. Details that could identify an individual need to be changed sufficiently, or deleted, to protect privacy. These details include, but are not limited to, names, account numbers, and addresses.
- Students should access suitable websites that provide information on credit cards and related calculations.
- The calculation of interest and other credit card charges varies between the different issuers of credit cards. Interest is typically charged for retail purchases, cash advances, balance transfers, and the amount still owing from the previous month. In this course, the same interest rate will be assumed to apply for all transaction types, and students will be required to calculate interest for amounts still owing from the previous month, cash advances, and retail purchases, but not for balance transfers.
- For interest calculations, simple interest is to be assumed. It is also to be assumed that interest is calculated on the daily outstanding account balance for each transaction and is applied at the end of the statement period. The daily interest rate is used for this calculation.

  Note: The daily interest rate is the annual percentage rate divided by 365, eg an annual rate of 16.5% is equivalent to a daily rate of $0.000\ 452$ (expressed as a decimal to three significant figures).
- Students should calculate the interest charged, closing balance, and minimum payment due given the annual interest rate, opening balance, at least three purchase transactions (item, date, amount), two cash advances (date, amount) and two or more repayments (date, amount).

Suggested applications

- Students create a credit card statement using a spreadsheet.
- Students compare, by making calculations, the costs associated with credit cards from different lenders. This should include consideration of the interest rates offered and fees.
- Students could use a spreadsheet or other appropriate technology to create a graphical representation comparing the use of two credit cards with different interest rates for the same purchases.

Strand: Data and Statistics

The collection and statistical analysis of data is of fundamental importance in society. Conclusions drawn from the statistical analysis of data are used to inform decisions made by governments, and, for example, in science, business and industry. It is important that students have a broad understanding of how data and statistics are used, as well as how statistics are misused, resulting in misconceptions about the appropriate use of statistics.

In the Data and Statistics Strand in the HSC Mathematics General 1 course, students use various data displays, and apply measures of location and measures of spread. They also investigate common statistical distributions and their associated properties, and misconceptions in statistical reasoning.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-2	analyses representations of data in order to make predictions
MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-7	develops and carries out simple statistical processes to answer questions posed
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

DS4CEC	Distributions
DS5CEC	Interpreting sets of data
DS6CEC	Working with statistics

Terminology

average	mean	Reversal Paradox
bell-shaped	measure of location	sample size
bi-modal distribution	measure of spread	scale
box-and-whisker plot	median	skewed distribution
distribution	mode	skewness
dot plot	normal distribution	standard deviation
double box-and-whisker plot	outlier	stem-and-leaf plot
double stem-and-leaf plot	population	symmetric distribution
histogram	quartile	two-way table
interquartile range	radar chart	uniform distribution
likelihood	range	unimodal distribution

Use of technology

Statistical software and spreadsheets should be used in the learning and teaching of the topic areas. Spreadsheets are widely used in the workplace and are a suitable tool for tabulating and graphing data and for calculating summary statistics.

Technology should be used to create frequency tables and statistical graphs, including box-and-whisker plots and radar charts.

The effect of outliers on the mean, median and mode of a data set can be investigated using appropriate technology.

Notes

Learning and teaching, and assessment materials, should use current information from a range of sources, including, but not limited to, surveys, newspapers, journals, magazines, bills and receipts, and the internet.

Real data should be used in the learning and teaching of the topic areas. Online data sources include the Australian Bureau of Statistics (ABS) website.

Suitable data sets for statistical analysis could include, but are not limited to, home versus away sports scores, male versus female data (eg for height), young people versus older people data (eg for blood pressure), population pyramids of countries over time, customer waiting times at fast-food outlets at different times of the day, and monthly rainfall for different cities or regions.

DS4CEC Distributions

In this topic, students learn to recognise and interpret properties of common statistical distributions.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-7, MG1H-9, MG1H-10

Content

Students:

- create a smooth curve to represent the general shape of a distribution
- recognise and describe in general terms different distributions of data, including normal, skewed, uniform, symmetric, unimodal and bi-modal distributions
- give examples of data sets that are normal, skewed, uniform, symmetric, unimodal and bi-modal
- determine the position of the mode(s) for different distributions from a graphical representation of a distribution
- recognise the shape of a distribution in various graphical forms, eg histogram, dot plot, stem-and-leaf plot, and box-and-whisker plot
- describe, for a given histogram, the shape of the distribution and how the shape relates to features of the associated population or sample
- identify the properties of data that are normally distributed, ie
  - the mean, median and mode are equal
  - if represented by a histogram, the resulting frequency graph (polygon) is ‘bell-shaped’
- solve problems involving interpretation of the standard deviation, where the value of the standard deviation is given.

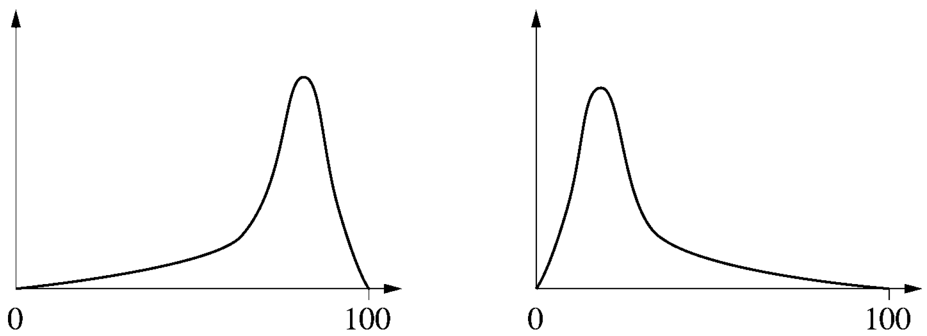
Considerations

- Graphical representations should be used to establish a general understanding of types of distributions.
- A general description of a distribution could include the type of distribution, and where applicable the number of modes, and the symmetry or skewness.
- Students should look at actual data in relation to giving examples of data sets that represent the various distribution types, eg data regarding average income per day for the world population in different years.
- The skewness of data can be described in practical terms by considering the location of the 'tail' of the graph of the distribution.
- When a die is thrown a large number of times, the distribution of the scores approaches a uniform distribution.

Suggested applications

- Discuss how many modes you would expect in the following distributions:
  - the heights of 3000 randomly selected adult women
  - the weight of gymnasts and sumo wrestlers
  - the distribution of the last digits of telephone numbers in a large city.

  Make a rough sketch for each distribution with clearly labelled axes.
- When constructing a graph of the weights of members of a rowing team, what effect does the inclusion of the cox generally have on the distribution?
- Determine the position of the mode in the following distributions:



DS5CEC Interpreting sets of data

The principal focus of this topic is the use of data displays, measures of location, and measures of spread to summarise and interpret one or more sets of data.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-7, MG1H-9, MG1H-10

Content

Students:

- identify measures of location: mean and median
- identify measures of spread: range, interquartile range and population standard deviation
- display data in double (back-to-back) stem-and-leaf plots
- display data in two box-and-whisker plots drawn on the same scale
- display two sets of data on a radar chart
- use multiple displays to describe and interpret the relationships between data sets
- interpret data presented in two-way table form, eg male/female versus exercise/no exercise
- compare summary statistics for two sets of data.

Considerations

- Interpretation of data sets includes deciding whether clustering is present, whether the shape of the display indicates any skewness, if there are outliers, or if there is any other tendency in the data.
- Students should be aware that the second quartile is also the median.
- Students should be presented with two-way tables that have row and column totals and some that do not. (See example below.)

Suggested applications

- The following example could be used to demonstrate the need for measures other than the range to describe spread:

  Samples of product were selected and weighed for two machines that produce chocolates with an advertised weight of 50 grams:

  Machine A: 45, 50, 50, 50, 50, 50, 50

  Machine B: 45, 45, 46, 46, 50, 50, 50

  The range of weights produced by both machines is 5 grams. Which machine produces chocolates that are more consistent in weight?
- The principal of a school of 500 students conducted a census to investigate how many students wore glasses:

	Always wear glasses	Sometimes wear glasses	Never wear glasses
Girls	52	A	170
Boys	38	59	147

  – Calculate the value of A.

  – How many boys sometimes wear glasses?

  – How many students always wear glasses?

  – What percentage of the girls always wear glasses?

  – What percentage of the students always wear glasses?

  – What percentage of the students who always wear glasses are girls?

DS6CEC Working with statistics

In this topic, students learn to recognise and interpret common misconceptions in statistical reasoning.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-7, MG1H-9, MG1H-10

Content

Students:

- describe common misconceptions in statistical reasoning, including:
  - misconceptions involving averages (often referred to as the Reversal Paradox)
  - interpreting percentage change for a small population compared to a larger population
  - misconceptions about sample size
  - 'the gambler's fallacy', eg after a run of heads when tossing a fair coin, a tail is more likely to occur on the next toss
- estimate the likelihood of events using a sample, based on how closely the sample matches the parent population.

Considerations

- The Reversal Paradox is often also referred to as Simpson's Paradox.
- Examples relating to 'interpreting percentage change for a small population compared to a larger population' include:

  In January 2007, there was a 400% decrease in road deaths in the Northern Territory compared to January 2006, whereas in NSW there was only a 39% decrease when comparing the same period. In fact, there were five road deaths in the Northern Territory in January 2006, compared to one road death in January 2007. This example shows that care needs to be taken when making comparisons between states/territories, as the base populations are very different.

  Source of data: *Australian Transport Safety Bureau*
- In relation to misconceptions about sample size, students need to be aware that a well-chosen sample can effectively represent a population even if the ratio of sample size to population size is small.
- A number of different situations relating to 'the gambler's fallacy' could be investigated, eg when tossing coins, it is common for people to believe that the sequence HTHHTTH is a more likely outcome than HHHHHTT. Such misconceptions are often used to determine fraudulent data.
- Explanations for the following research findings could be discussed:
  - children with bigger feet spell better
  - nations that add fluoride to water have higher incidences of cancer
  - there are more motor vehicle accidents at moderate speed than at high speed.

Suggested applications

- The Reversal Paradox could be illustrated using the following example:

  Two cricketing brothers challenged each other to have the better overall batting average over the next two Test series. In the first series, John scored 500 runs for 10 outs, an average of 50, while James scored 270 runs for 6 outs, an average of 45. In the second series, John scored 320 runs for 4 outs, an average of 80, while James scored 700 runs for 10 outs, an average of 70. John thought he had won because he had a higher average than James in each series.

	John	James
First series	$500 \div 10 = 50$	$270 \div 6 = 45$
Second series	$320 \div 4 = 80$	$700 \div 10 = 70$
Overall	$820 \div 14 = 58.6$	$970 \div 16 = 60.6$

  How is it possible that John could have a better average in each of the two Test series, but a lower average overall?

Strand: Measurement

Perimeter, area and volume have many practical applications, including in building, surveying and manufacturing. The study of Measurement is important in developing students' awareness of the broad range of applications of measurement concepts in everyday life, including, for example, in storage and packaging.

In the Measurement Strand in the HSC Mathematics General 1 course, students extend and apply their knowledge, skills and understanding in relation to area and volume to more complex figures.

Outcomes addressed

A student:

MG1H-4	analyses simple two-dimensional and three-dimensional models to solve practical problems
MG1H-5	interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

MM4CEC Further applications of area and volume

Terminology

annular cylinder	composite figure	sector
annulus	irregular	Simpson's rule
approximation	perimeter	sphere
area	prism	surface area
closed cylinder	quadrant	volume

Use of technology

Students should be given the opportunity to use suitable software to create both two-dimensional (2D) and three-dimensional (3D) drawings.

The internet could be used as a source of suitable maps and plans for the estimation of areas using Simpson's rule.

Notes

Students should be encouraged to 'estimate and check' to determine if results are reasonable. This is a skill that should be reinforced throughout the Measurement Strand.

Learning and teaching should be supported through access to industry-standard house plans and maps.

Knowledge, skills and understanding developed in Measurement in the HSC Mathematics General 1 course is applied, consolidated and extended in the Focus Studies in the course.

Vocational applications of the concepts studied should be investigated, eg the use of Simpson's rule in surveying.

MM4CEC Further applications of area and volume

The principal focus of this topic is to extend the work commenced in MM2 *Applications of perimeter, area and volume* to include area of composite shapes, surface area, and volume of more complex figures.

Outcomes addressed

MG1H-4, MG1H-5, MG1H-10

Content

Students:

- calculate the area of a circle
- calculate the area of an annulus from a given diagram
- calculate the area of a sector as a fraction of a circle
- calculate areas of composite figures constructed from squares, rectangles, triangles and circles
- estimate an area using a single application of Simpson's rule over three equally spaced points ($A \approx \frac{h}{3}(d_f + 4d_m + d_l)$)
- calculate the surface area of a cube, rectangular prism, sphere, and closed cylinder
- calculate the volume of a right prism with an irregular base, where the area of the base is known
- calculate the volume of a hollow annular cylinder
- solve practical area, surface area and volume problems
- estimate and check to determine if results are reasonable.

Considerations

- The angles for sectors of a circle should be limited to factors of 360°, eg 180°, 120°, 90°, 60°, 45°, 30°, 15°, etc.
- For annulus calculations, diagrams could have either the radii or diameters labelled.
- Teachers could extend the use of Simpson's rule to two or more applications.
- Manipulation of nets could be used to enhance understanding of the surface area of prisms and cylinders.

Suggested applications

- How many cubic metres of concrete are required for a driveway that is 4.5 metres wide, 12 metres long and 100 millimetres thick?
- Design the shape and dimensions of a container that would have a particular capacity, given the purpose and use of the container.
- Design cost-effective packaging, eg groups of students are given four table-tennis balls and need to design two different boxes to package them. Students should then determine the better of the two designs in terms of minimisation of material used.
- As designers, students are given a square piece of metal of side length 2 metres from which to design an open rectangular water tank. The volume of water that the tank will hold depends on the size of squares cut from each of the four corners of the piece of metal. Students choose a scale, make models of tanks and find the volume of water that they can hold. They could graph results and determine when the volume is the greatest. Students could also be asked to consider what happens if the side of the original square is doubled.
- Students calculate the surface area and volume of sphere-shaped sweets or cylinder-shaped sweets, eg marshmallows. Class results could be combined to form a data set for statistical analysis.

  Discussion would include:

  – What assumptions are being made about the shape of the sweet?

  – How can a reasonably accurate measure for the radius be obtained?

  – How accurate will the results be?

  – Are the results normally distributed?

  – What claims could the manufacturer make about the surface area and volume of that particular product?

Strand: Probability

In developing an appreciation of the broad range of applications of probability in everyday life, students should be aware of its use in such fields as insurance and quality control in industry. The use of probability concepts assists in making informed decisions in relation to games of chance.

In the Probability Strand in the HSC Mathematics General 1 course, students identify the outcomes expected from simple experiments, compare them with experimental results, and establish the probabilities of outcomes and events.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-2	analyses representations of data in order to make predictions
MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

PB2CEC	Multistage events and applications of probability

Terminology

arrangement
expected number
experimental result
financial expectation
financial gain
financial loss
multistage event
probability tree diagram
sample space
simulation
theoretical probability
tree diagram
trial
two-stage event
with replacement
without replacement

Use of technology

Students should design and use a spreadsheet to simulate large numbers of trials. The graphing facilities of a spreadsheet could be used to investigate the results of a simulation.

Online probability simulations could be used for the investigation of large numbers of trials.

Notes

Formulae involving factorial notation are not required in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.

'Probability tree diagrams' can be developed as a shorthand for tree diagrams in which every branch represents an equally likely event. Using the multiplication principle and a tree diagram in which every branch represents an equally likely event establishes the conceptual background for multiplying along branches of a probability tree diagram.

Simulations of large numbers of trials should be used to make comparisons with theoretical probabilities.

PB2CEC Multistage events and applications of probability

The focus of this topic is on counting the number of outcomes for an experiment and the number of ways in which an event can occur, and identifying the outcomes expected from simple experiments and comparing them with experimental results. The probability of particular outcomes and events can then be established.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-10

Content

Students:

- multiply the number of choices at each stage to determine the number of outcomes for a multistage event
- establish that the number of ways in which n different items can be arranged in a line is $n(n-1)(n-2)\ldots\times 1$,eg the number of arrangements of four different items is $4\times 3\times 2\times 1=24$; the number of arrangements of three different items is $3\times 2\times 1=6$
- construct and use tree diagrams to establish outcomes for a simple two-stage event
- use probability tree diagrams to solve problems involving two-stage events
- calculate the expected number of times a particular event would occur, given the number of trials of a simple experiment, by establishing the theoretical probability of the event and multiplying by the number of trials
- compare the expected result with an experimental result
- calculate the financial expectation by multiplying each financial outcome by its probability and adding the results together.

Considerations

- Multistage events considered should involve both *with replacement* and *without replacement*.
- Formulae involving factorial notation are not required in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.
- Selecting cards from a pack of cards, *with replacement* and *without replacement*, could be used to investigate two-stage experiments.
- A financial loss is regarded as negative.
- When playing games of chance, any entry fee into a game is considered a financial loss.

Suggested applications

- Determine the total number of different outcomes in a game in which five numbers are chosen from 20 different numbers.
- How many different ways are there of answering a four-question true/false test? Check by listing the possible responses.
- In how many ways can the names of three candidates be listed on a ballot paper? What is the probability that a particular candidate's name will be at the top of the paper? Check by listing.
- Determine whether it is better to buy 10 tickets in one lottery, or one ticket in each of 10 lotteries.
- Investigate different strategies for playing party games such as 'Greedy Pig'.

  The aim of 'Greedy Pig' is to finish with the highest score. The score is determined by adding the number rolled on the uppermost face of a die to those previously rolled. Students all stand. After two rolls they may sit down if they wish to keep their current score. The game ends when a 6 is rolled. The score of those who are standing at that time becomes zero. The game should be played numerous times to allow the students to change their playing strategy.
- Paul plays a game involving the tossing of two coins. He gains $\$5$ if both coins show heads and $\$1$ if they show a head and a tail, but loses $\$6$ if they both show tails. What is his financial expectation for this game?
- Conduct simulations to model events, eg tossing a coin to represent the sex of children born, with a head indicating a boy and a tail indicating a girl.

Strand: Algebra and Modelling

Algebra is fundamental to generalisation in mathematics and the solution of problems across subject areas. It is an essential tool in problem-solving through the solution of equations, the graphing of relationships, and modelling with functions.

In the Algebra and Modelling Strand in the HSC Mathematics General 1 course, students develop and apply algebraic skills and techniques to interpret and use linear and non-linear mathematical models in a range of vocational and other practical contexts.

Outcomes addressed

A student:

MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

AM3CEC	Further algebraic skills
AM4CEC	Modelling with functions

Terminology

break-even	index	minimum value
coefficient	index laws	modelling
constant	interpolate	parabola
denominator	linear function	point of intersection
expression	magnitude	profit
extrapolate	maximum value	quadratic function

Use of technology

Students should have access to appropriate technology in order to create graphs of linear and quadratic functions.

The ‘zoom’ feature on a spreadsheet or graphing calculator could be used to explore graphs and to find the points of intersection of graphs, eg in ‘break-even’ analysis.

Notes

Algebraic skills should be developed through the use of formulae and algebraic expressions from vocational and other practical contexts.

The Focus Studies in the course provide a range of opportunities for students to apply knowledge, skills and understanding developed in the Algebra and Modelling Strand.

AM3CEC Further algebraic skills

In this topic, students develop algebraic skills and techniques that have applications in work-related and everyday contexts. These skills and techniques include the development of competency in finding the values of pronumerals following substitution in algebraic formulae.

Outcomes addressed

MG1H-3, MG1H-9, MG1H-10

Content

Students:

- establish and apply index laws $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $\left(a^m\right)^n = a^{mn}$
- apply index laws to simplify algebraic expressions, eg $(x^3)^4$, $4x^2 \times 5x^3$, $15w^7 \div 5w^3$, $3p^2(p-2)$
- solve equations, including equations where solution involves the removal of brackets and equations with an unknown in the denominator, eg $3(a+7)=28$, $15=\dfrac{45}{h}$, $\dfrac{t}{15}-50=175$
- solve for a linear term in an equation following substitution into a mathematical formula from a vocational or other context,

  eg if $B=\dfrac{m}{h^2}$, find m given that $B=23, h=1.63$.

Considerations

- Teachers may find it necessary to revise skills developed in AM1 *Algebraic manipulation*, including:
  - identification and generalisation of simple linear number patterns
  - addition and subtraction of like terms
  - expansion and simplification of algebraic expressions
  - factorisation of algebraic expressions
  - multiplication of algebraic terms
  - substitution into algebraic expressions.

Suggested applications

- Substitution into formulae should involve the use of a variety of vocational and other practical formulae, including formulae involving variables given in words, eg

  $D = \frac{kA}{70}$ $\qquad$ $Child\ dosage = \frac{age\ (in\ months) \times adult\ dosage}{150}$

  $Heart\ rate_{\max} = 205.8 - (0.685 \times age)$.

AM4CEC Modelling with functions

This topic focuses on modelling using linear and quadratic functions. Students learn to apply and graph these functions in vocational and other practical contexts.

Outcomes addressed

MG1H-3, MG1H-9, MG1H-10

Content

Students:

- generate tables of values for linear functions (including for negative values of x)
- graph linear functions for all values of x with pencil and paper, and with graphing software
- interpret the point of intersection and other important features of given graphs of two linear functions drawn from practical contexts, eg break-even point
- generate tables of values for quadratic functions of the form $y = ax^2$ and $y = ax^2 + c$ (including negative values of a and x)
- graph quadratic functions with pencil and paper, and with graphing software
- explain the effect of changing the magnitude of a and changing the sign of a
- explain the effect of changing the value of c
- identify the maximum and minimum values of a quadratic function from a prepared graph based on a practical context
- recognise the limitations of models when interpolating and/or extrapolating
- use linear and quadratic functions to model physical phenomena.

Considerations

- Students should be able to solve break-even problems graphically in questions emphasising the break-even point, the profit zone and the loss zone, and interpretation of the y-intercept.
- Students should construct tables of values and create graphs of $y = x^2$ and $y = -x^2$.
- Graphing software can be used to generate graphs of linear and quadratic functions.
- In modelling physical phenomena, functions and graphs should involve only positive values of the independent variable and zero.

Suggested applications

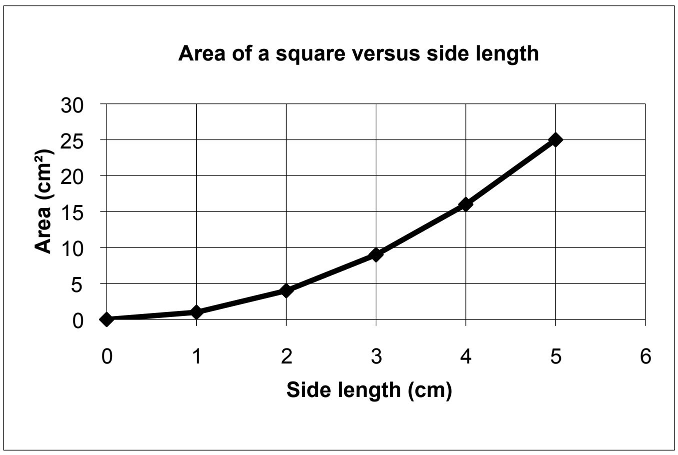
- Students could draw and interpret a variety of distance/time graphs.
- The following scenario is depicted in the graph below: Martha sells muffins for $2.50 each. It costs $1 to make each muffin and $300 for the equipment needed to make the muffins.

Typical questions based on the graph could include:

– How many muffins need to be sold to ‘break even’?

– How much profit is made if 400 muffins are sold?

- The graph shows the relationship between the area of a square and the length of its side. Use the graph to find the side length of a square with area 16 cm^2.



Focus Study: Mathematics and Design

Mathematical concepts are of fundamental importance in all fields involving design, including architecture, engineering, art, decoration, textiles, jewellery and music. Design brings together mathematical skills in number, geometry and measurement, including trigonometry, to develop drawings, plans, models and constructions.

In the Mathematics and Design Focus Study, students apply, and develop further, knowledge, skills and understanding in Measurement, and Algebra and Modelling, to practical design contexts.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-4	analyses simple two-dimensional and three-dimensional models to solve practical problems
MG1H-5	interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

FSDe1CEC	Scale drawings and house plans
FSDe2CEC	Design

Terminology

area
dimension
elevation view
golden ratio
intersect
irregular
line symmetry
non-regular tessellation
parallel
perpendicular
perspective
plan view
plane
polygon
Pythagoras
radial symmetry
regular
regular tessellation
rotation
rotational symmetry
scale factor
semi-regular tessellation
similarity
tessellation
three-dimensional (3D)
trigonometry
two-dimensional (2D)
volume

Use of technology

Students should use geometry software (eg GeoGebra freeware available on the internet) to construct two-dimensional (2D) and three-dimensional (3D) drawings and designs. Teachers can also use this software to demonstrate the geometrical properties of polygons.

Notes

During lesson time, students require access to geometrical instruments, including a pair of compasses, protractor and ruler, as well as house plans and other scale drawings.

FSDe1CEC Scale drawings and house plans

In this topic, students interpret and use house plans, designs and maps in the calculation of a range of measurements and the solution of related problems.

Outcomes addressed

MG1H-3, MG1H-4, MG1H-5, MG1H-9, MG1H-10

Content

Students:

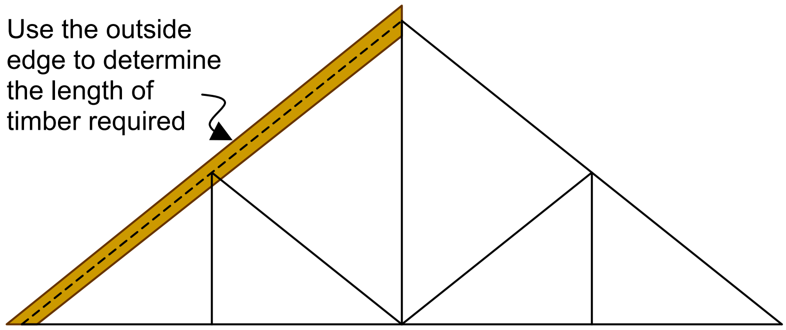
- sketch plan views and elevation views of three-dimensional objects
- recognise parallel, perpendicular and intersecting lines, in the context of two-dimensional and three-dimensional representations of houses and buildings
- define and recognise planes in three-dimensional space in the context of three-dimensional representations of houses and buildings
- interpret common symbols and abbreviations on house plans
- use the scale on a plan, design or map to calculate actual dimensions, and vice versa
- interpret plan views and elevation views to obtain internal dimensions of rooms
- calculate area and volume based on information on a plan
- apply right-angled triangle trigonometry and Pythagoras' theorem to solve problems based on plans, including finding the pitch of a roof.

Considerations

- Three-dimensional (3D) representations of houses and buildings could include isometric view plans. Students should be able to identify parallel planes based on two-dimensional (2D) plans, ie the plan view and the various elevations. The floor and ceiling of a house, for example, are parallel planes.
- The required dimensions on a plan or diagram of a roof truss may be determined by a student in a variety of ways, including the dimensions are provided on the plan (or diagram), the dimensions are calculated using the scale of the plan, the dimensions are measured from the plan, or Pythagoras' theorem or trigonometry is applied to calculate an unknown dimension.
- Calculations involving roof trusses should be limited to Howe, Double Howe, Fink and Double Fink trusses.
- Most problems involving calculations of lengths will be simplified by ignoring the thickness of the timber and simply using the length of the lines in a line diagram.
- Interpretation of a house plan and making calculations using information on the plan need to include the calculation of internal dimensions of rooms.
- Calculation of area and volume based on information on a plan needs to include, for example:
  - finding the area of a house to be carpeted and the cost of purchasing the carpet
  - calculating the area to be painted and the cost of painting a room in a house
  - calculating the volume of the rooms in a house and using a table to determine the appropriate-sized air conditioner for the house.

Suggested applications

- Some questions may require the calculation of the length of timber required to build a truss based on the outside (ie the longer) dimensions of the pieces of timber. This is a more accurate approach to calculating the length of timber required, eg calculate the total cost of the timber in the Howe truss below.



- A more sophisticated exercise might involve calculating the number of lengths required and the cost, given that timber is sold in set lengths. Students could also be asked to allow a certain amount for wastage, eg 5%.
- Investigate styles of roofs for different climatic conditions.

FSDe2CEC Design

In this topic, students learn to identify and apply various concepts important in design, including scale factors, similarity and symmetry. In producing sketches, constructions and designs, they use geometrical instruments as well as computer software.

Outcomes addressed

MG1H-1, MG1H-3, MG1H-4, MG1H-5, MG1H-9, MG1H-10

Content

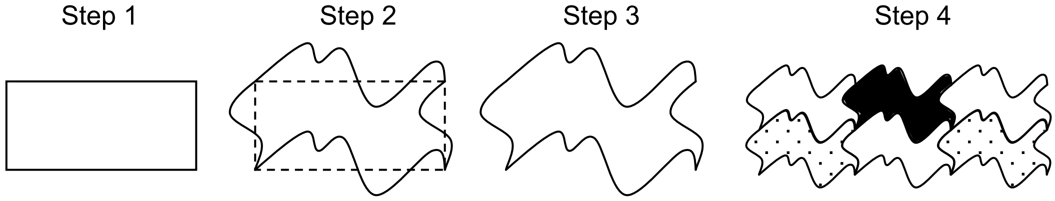
Students:

- enlarge and reduce plane shapes by a specified scale factor, using a ruler and a pair of compasses
- recognise and apply similarity to calculate lengths and areas of regular and irregular plane shapes
- sketch common three-dimensional objects, including rectangular and triangular prisms, cylinders, pyramids and cones
- recognise parallel, perpendicular and intersecting lines in two-dimensional shapes and three-dimensional objects
- identify line and radial symmetry (rotational symmetry) in common mathematical shapes, designs, artworks and architecture
- create, with the aid of a ruler, examples of simple perspective drawings
- identify examples of the golden ratio in art and design by appropriate calculation
- recognise and construct simple tessellations of three kinds: regular, semi-regular and non-regular
- construct a simple design by hand and with technology, using common geometrical shapes.

Considerations

- Students should recognise the application of perspective in works of art, eg art from the Renaissance period.
- Students should identify examples of architecture that are mathematically influenced, with a brief explanation of the geometrical features evident in the design.
- Calculations using the side lengths of rectangular shapes, such as windows and doors, could be performed to check if the rectangle represents a golden rectangle.
- Students should recognise that there are exactly three regular tessellations of the two-dimensional plane. There are only three because the size of the interior angles of the polygon must be a factor of $360°$, so that the polygons cover the plane without leaving gaps or overlaps. The three regular tessellations of the two-dimensional plane are the tessellations of the equilateral triangle, the square, and the regular hexagon.
- Non-regular tessellations may be constructed by adding and subtracting sections from the edges of a regular tessellation, such as in the diagram below.

Construction of a non-regular tessellation



Suggested applications

- Students find examples of geometrical shapes and patterns used in interior design.
- Use isometric dot paper to construct sketches of three-dimensional objects.
- Investigate the use of vanishing points in perspective drawings.
- Students identify examples of the golden ratio in art, nature and architecture.
- Students create a tessellating pattern using either pen and paper or geometry software.
- Students verify the relationship in a tessellation that $(a-2)(b-2)=4$, where a is the number of sides of a polygon and b is the number of sides of the polygon meeting at a vertex, eg for a tessellation of equilateral triangles, $a=3$ and $b=6$.
- Investigate the number of points of stars on flags of countries of the world and how the stars might be constructed, eg the Commonwealth Star on the Australian flag has seven points.
- Students use geometry software to construct two-dimensional (2D) and three-dimensional (3D) drawings.

Focus Study: Mathematics and Household Finance

Sound skills in the management of household expenses are necessary in ensuring that the occupants are provided with appropriate levels of food, water, electricity, etc. The development of students' awareness of financial aspects of household management needs to include the development of understanding of the costs of renting or owning accommodation and the costs of day-to-day living, and the development of skills in the management of income and spending.

In the Mathematics and Household Finance Focus Study, students apply, and develop further, knowledge, skills and understanding in Financial Mathematics, Measurement, and Algebra and Modelling to contexts related to household finance.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-2	analyses representations of data in order to make predictions
MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-4	analyses simple two-dimensional and three-dimensional models to solve practical problems
MG1H-5	interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units
MG1H-6	makes informed decisions about financial situations likely to be encountered post-school
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

FSHo1CEC	Accommodation costs: buying and renting
FSHo2CEC	Costs of running a household, maintenance and repairs

Terminology

budget
consumption
conveyancing
energy rating
insurance
levy
lineal
maintenance
mortgage
public liability
reducing balance loan
renovation
rental bond
repayment
stamp duty
strata levy
utility

Use of technology

A spreadsheet can be used to create budgets, model home loans, and produce relevant graphical representations.

Students use online resources to compare mortgage application fees and interest rates, and to make calculations regarding repayments.

Online resources can be used to investigate energy-efficient appliances (eg shower heads and light globes) and to create visual representations of the savings.

Online calculators can be used to investigate energy efficiency as it relates to the use of heating and cooling appliances.

Students should use geometry software (eg GeoGebra freeware available on the internet) to construct two-dimensional and three-dimensional drawings and designs.

Notes

Learning and teaching, and assessment, should draw on and integrate the mathematics in Strands within the course, eg Financial Mathematics, Data and Statistics, Measurement, and Algebra and Modelling.

Students require access to current information from a range of sources, including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

Students need to have access to examples of utility bills during lesson time.

The decoration concepts presented in this Focus Study provide a creative application of mathematics for those students who enjoy design and/or construction. Some teachers may wish to combine aspects of this Focus Study with some of the knowledge, skills and understanding developed in the Mathematics and Design Focus Study.

FSHo1CEC Accommodation costs: buying and renting

In this topic, students calculate costs involved in purchasing and renting houses and units, and use tables and graphs that they have constructed in relation to home loans. Students also investigate changes in housing and renting costs over time and the effect of changes in interest rates.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-5, MG1H-6, MG1H-9, MG1H-10

Content

Students:

- calculate the affordability of accommodation based on income, using generalised rules such as monthly payments should not be greater than a certain percentage of income; and the amount to be borrowed should not exceed a certain multiple of the annual household income
- calculate the costs involved in purchasing a house or unit, including stamp duty, mortgage application fees, and conveyancing
- calculate the costs involved in renting a house or unit, including the cost of a rental bond
- use published tables from financial institutions to determine monthly repayments on a reducing balance home loan
- use monthly repayment tables for a home loan to calculate the total amount to be repaid and the total interest to be paid
- construct a graph of changes in interest rates over a given period of time
- construct tables and graphs using online loan calculators to investigate the implications of changes to interest rates, changes in minimum repayments, and changes in the length of the total repayment period for a loan.

Considerations

- Different home loans should be compared, including variables such as the interest rate, loan establishment costs, and early exit fees.
- When investigating changes in minimum repayments or changes in interest rates, etc, all other variables should be kept constant.
- Some online home loan calculators give graphical and tabular representations of changes to variables.

Suggested applications

- Select some rental properties from the local paper or online and, assuming that accommodation payments should not be more than 30% of after-tax income, calculate the income required to rent the property. Investigations could be extended to include different suburbs, types of accommodation, or sharing of accommodation.
- For a selected property, calculate a 10% deposit and the amount of the balance to be borrowed. If the amount to be borrowed should not exceed 2.5 times the annual income of the household, calculate the required annual income to purchase the property.
- Use an online home-loan calculator to calculate repayments and total interest paid on different-sized loans.
- Online calculators could be used to hold the interest rate constant and to investigate the effect of paying higher than the minimum monthly repayment for the duration of the loan, and hence the difference in the total amount of interest paid.
- Use a prepared graph of ‘amount outstanding’ versus ‘repayment periods’ to determine when a particular loan will be half-paid.
- Students could use an online calculator to investigate the effect of changes in interest rates on the monthly repayment for a loan. The results could be presented in a table or graph.
- Compare any changes in the cost of rental property in a particular suburb to that of buying similar property in that suburb. Graphs such as line graphs or radar charts could be constructed.
- Students could compare the cost of renting similar properties in different locations.

FSHo2CEC Costs of running a household, maintenance, and repairs

The principal focus of this topic is the calculation and comparison of household running, maintenance and repair costs. Students also investigate ways to reduce household expenditure and minimise wastage.

Outcomes addressed

MG1H-2, MG1H-3, MG1H-4, MG1H-5, MG1H-6

Content

Students:

- compare the costs of various insurances, including public liability, building, contents, income protection, and personal insurance
- investigate body-corporate and strata levies
- read and interpret common household bills, including bills for electricity, gas, telephone, council rates, land tax, water, and body-corporate and strata levies
- perform calculations based on information contained in common household bills
- investigate ways in which household expenditure can be reduced, eg efficient shower heads, and more efficient light globes and appliances
- calculate the costs of common repairs carried out by tradespeople, given the hourly rate and the cost of materials
- perform calculations for home additions, renovations, repairs and maintenance
- construct a scale diagram of a room
- calculate the cost of repainting rooms based on the calculation of the area of the walls and ceilings
- calculate the amount of floor covering required for a room
- prepare a budget reflecting the costs of running a household.

Considerations

- Students need access to examples of household bills, such as bills for electricity, water, gas and telephone.
- Cost of repairs by tradespeople should include consideration of the cost of out-of-hours ('emergency') repairs.
- Published charts or online calculators should be used to make accurate calculations of quantities of materials required for decoration or renovation.
- Expenses for building additions are normally quoted per square metre. Students should be aware of other costs involved in building additions, eg engineer's fees, architect's fees, and council fees.
- The dimensions of the room and the width of the carpet should be considered when deciding in which direction to lay the carpet in order to minimise wastage and the number of joins.
- Examples of floor covering materials should include materials that use lineal (carpeting) and square (tiling) measures.

Suggested applications

- Use the notice above to calculate the residential rate and the total payable.
- Students could write a proposal to redecorate a room, including calculations of the cost of painting and furnishing the room, as well as the total cost.
- Students use electronic drawing tools to construct two-dimensional and three-dimensional drawings.
- Students plan and cost a decoration or renovation project, including calculations of the amount and cost of materials required, eg the amount and cost of materials required for making a mosaic pot, a patchwork quilt, or a metal sculpture.
- Construct a scale diagram of a room and use the diagram to calculate the amount of paint required to repaint the walls and ceiling. (Online calculators are available.)

Focus Study: Mathematics and the Human Body

The study of the human body and related health issues involves mathematical knowledge, skills and understanding, including in relation to heart rate, blood pressure, body measurements and lung capacity.

In the Mathematics and the Human Body Focus Study, students apply, and develop further, knowledge, skills and understanding in Data and Statistics, Measurement, Probability, and Algebra and Modelling to contexts related to the human body.

Outcomes addressed

A student:

MG1H-1	uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts
MG1H-2	analyses representations of data in order to make predictions
MG1H-3	makes predictions about everyday situations based on simple mathematical models
MG1H-4	analyses simple two-dimensional and three-dimensional models to solve practical problems
MG1H-5	interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units
MG1H-8	solves problems involving uncertainty using basic counting techniques
MG1H-9	chooses and uses appropriate technology to organise information from a range of practical and everyday contexts
MG1H-10	uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

FSHu1CEC	Blood
FSHu2CEC	Body measurements
FSHu3CEC	Lung capacity

Terminology

biometric data	Estimated Vital Lung Capacity	strength of association
blood type	line of fit	systolic
compatibility	linear	Targeted Heart Rate
correlation coefficient	lung capacity	theoretical value
diastolic	ordered pairs	Theoretical Vital Lung Capacity
estimated value	scatterplot	trend

Use of technology

Appropriate technology should be used to construct, and determine the equation of, a line of fit, and to calculate correlation coefficients.

A spreadsheet could be created to calculate measures of Estimated Vital Lung Capacity.

The internet should be used as a source of up-to-date information, eg data and statistics on blood collection, body measurements, and the lung capacities of athletes.

Calculators can also be accessed on the internet for the calculation of Theoretical Vital Lung Capacity.

Teachers should demonstrate the 'trendline' function on a spreadsheet and then have students explore the function with their own sets of data.

Notes

Learning and teaching, and assessment, should draw on and integrate the mathematics in Strands within the course, eg Data and Statistics, Measurement, Probability, and Algebra and Modelling.

Students require access to current information from a range of sources, including, but not limited to, newspapers, journals, magazines, real bills and receipts, and the internet.

There are many opportunities for the application of algebraic skills in this Focus Study. A range of formulae, other than those listed in the content, could be used, eg formulae for Body Surface Area and Targeted Heart Rate.

This Focus Study provides students with special interest in the health sciences, PDHPE, or Visual Arts the opportunity to explore aspects of mathematics involved in those areas.

FSHu1CEC Blood

In this topic, students interpret charts, construct graphs, and perform a range of calculations in relation to blood and heart rate. They identify trends and make predictions based on their calculations.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-4, MG1H-5, MG1H-8, MG1H-10

Content

Students:

- describe heart rate as a rate expressed in beats per minute
- measure and graph a person's heart rate over time under different conditions, eg at rest, during exercise, and after exercise
- identify mathematical trends in heart rate over time under different conditions
- calculate the total number of heart beats over a given time under different conditions
- calculate Targeted Heart Rate ranges during training
- express blood pressure using measures of systolic pressure and diastolic pressure
- measure blood pressure over time and under different conditions
- read a blood pressure chart and interpret the 'healthiness' of a reading
- interpret data in a blood compatibility chart as an alternative presentation of data in a two-way table
- predict, by calculation, the number of people of each blood type in a population given the percentage breakdowns
- predict, by calculation, the expected number of people of a particular blood type in a population.

Considerations

- Blood pressure graphed over time involves the introduction of three variables. Prepared graphs could be used if measuring equipment is unavailable. The appropriateness of different types of graphs to display this information should be discussed.

Suggested applications

- Measure and record heart rate under different conditions, eg heart rate during different exercises (could include both aerobic and non-aerobic activities), recovery rate after exercise, relationship between a person's resting heart rate and heart rate after exercise, or recovery time after exercise.

  Discussion could include:

  - whether the relationships are linear or non-linear
  - expected differences between resting heart rate in non-stressful conditions versus in stressful conditions
  - other measures (factors) of health that may affect heart rate.
- Students calculate ranges of Maximum Heart Rates.
- The amount of blood pumped by the heart over time, given different exercise conditions, could be investigated.
- Students investigate graphs of heart rate and temperature over time (as used in hospitals).
- Students investigate:
  - blood donation rates in different populations
  - trends in donation rates and usage at different times of the year.
- The percentages of different blood types in a range of different countries could be investigated.
- Students make predictions about the use of blood collected, given usage statistics. They could also perform calculations in relation to the need for ongoing blood donation, given the shelf-life of blood products.

FSHu2CEC Body measurements

In this topic, students construct scatterplots and lines of fit and use them to explore relationships and make predictions. They use technology to calculate correlation coefficients and interpret the strength of association of variables.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-4, MG1H-5, MG1H-9, MG1H-10

Content

Students:

- investigate biometric data obtained by measuring the body and by accessing published data
- plot ordered pairs of body measurement data onto a scatterplot by hand and by using appropriate technology
- recognise patterns in a scatterplot of body measurements, eg
  - whether the points appear to form a mathematical pattern
  - whether the pattern appears to be linear
- construct a line of fit and determine the equation, by hand and by using appropriate technology
- use the equation of a line of fit to make predictions about body measurements
- recognise the practical limitations of the equation of a line of fit
- calculate correlation coefficients for different body measurements using appropriate technology (students are not required to calculate correlation coefficients by hand)
- interpret the strength of association for different body measurements using a given correlation coefficient
- interpret the sign of a given correlation coefficient.

Considerations

- The biometric data obtained should be used to construct lines of fit and determine their equations, and to determine correlation coefficients (using a spreadsheet or other appropriate technology). Scatterplots drawn could include scatterplots of height versus arm-span, height versus weight, hip-height versus stride length, hand-span versus height, foot-length versus height, or hand-span versus foot-length. Gender could be added, where appropriate, as an additional variable. For example, male and female data for arm-span versus height could appear on the same scatterplot, but have different lines of fit.

Suggested applications

- Students could measure body dimensions such as arm-span, height and hip-height, as well as length of stride. It is recommended that students have access to published biometric data to provide suitable and realistic learning contexts. Comparisons could be made using parameters such as age or gender.
- Predictions could be made using the line of fit, eg predicting a person's height based on their arm-span or arm-length. Students should assess the accuracy of the predictions by measurement and calculation in relation to an individual not in the original data set, and by the value of the correlation coefficient.
- Forensic science formulae for the prediction of human height based on the length of the radius bone could be investigated and a comparison made between the students' actual measurement and the value calculated from the formulae below:

  $$Height_{male} = 80.405 + 3.650r$$

  $$Height_{female} = 73.502 + 3.876r$$

  where $Height$ and the length of the radius bone (r) are in centimetres. The radius bone is the exterior bone of the lower arm, from wrist to elbow when standing in a palm-forward position. (Only an approximate estimate will be possible for a live person.)
- Students investigate Leonardo da Vinci's Vitruvian theory.
- Students could model the following: 'After the age of 30, the height of a person begins to decrease at the rate of approximately 0.06 cm per year.' Representations in graphical and/or equation form can then be used to make predictions about a person's height at a given age.

  Other body measurements could be taken and used to investigate the Golden Ratio phi (φ), eg the ratio of the length of the upper arm to the length of the hand, or the measurement from navel to toe compared to the measurement from head to toe. The Golden Ratio could be introduced or further explored in the Mathematics and Design Focus Study in this course.

FSHu3CEC Lung capacity

In this topic, students recognise lung capacity as a volume and perform a range of related calculations. They compare estimated and theoretical values where relevant.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-4, MG1H-5, MG1H-10

Content

Students:

- recognise lung capacity as a volume by obtaining values for Estimated Vital Lung Capacity by practical means, and statistically analyse the values
- calculate a person's Theoretical Vital Lung Capacity (in litres)
- convert lung capacity from cubic centimetres to litres

  $1\text{ cm}^3 = 1\text{ mL}$ ($= 0.001$ L), $1000\text{ cm}^3 = 1$ L
- compare values of Estimated Vital Lung Capacity with theoretical values.

Considerations

- It should be noted that Theoretical Vital Lung Capacity is expressed in litres, while Estimated Vital Lung Capacity is expressed in cubic centimetres.

  Theoretical Vital Lung Capacity is the measure in litres of the amount of air a person's lungs should hold based on their age and height.

  A person's Theoretical Vital Lung Capacity (in litres) should be calculated using the formula

  $$Theoretical\,Vital\,Lung\,Capacity = 0.041 \times Height\,(cm) - 0.018 \times age - 2.69$$

  where $Height$ is measured in centimetres and age is measured in years.

Suggested applications

- Students calculate practical measures of lung capacity, using a balloon.

  This will require the assumption that the balloon is a sphere and the use of the formula for the volume of a sphere: $V = \frac{4}{3}\pi r^3$.

  The limitations of this method of estimation should be explored, eg

  – How can an accurate measure of the diameter of the balloon be obtained?

  – Is this method of estimation likely to overestimate or underestimate the vital lung capacity?

  – What issues are likely to arise if the experiment is repeated with the same balloon or a balloon of a different colour?

  Alternative methods could be explored, eg graphical methods for the calculation of the volume of a balloon.

- Explain why the variables height and age might be important factors in Theoretical Vital Lung Capacity.
- Explain the difference between Estimated Vital Lung Capacity and Theoretical Vital Lung Capacity, as well as any limitations.
- Compare calculated values obtained using both Estimated Vital Lung Capacity and Theoretical Vital Lung Capacity.
- Students could identify and investigate factors affecting lung capacity, including age, gender, and medical conditions such as asthma, smoking, and level of fitness. Investigations could include the collection, recording and interpretation of data to support a hypothesis.
- Students investigate the lung capacity of elite athletes in various sports.

Focus Study: Mathematics and Personal Resource Usage

The efficient use of energy in the home and the workplace is a major and increasing area of concern for individuals and communities. Householders can reduce their energy bills and contribute to the sustainability of resources by employing strategies designed to reduce waste and promote recycling.

In the Mathematics and Personal Resource Usage Focus Study, students apply, and develop further, their knowledge, skills and understanding in Financial Mathematics, Data and Statistics, Measurement, and Algebra and Modelling to contexts involving personal resource usage.

Outcomes addressed

A student:

MG1H-1 uses mathematics and statistics to evaluate and construct arguments in a range of familiar contexts

MG1H-2 analyses representations of data in order to make predictions

MG1H-3 makes predictions about everyday situations based on simple mathematical models

MG1H-4 analyses simple two-dimensional and three-dimensional models to solve practical problems

MG1H-5 interprets the results of measurements and calculations and makes judgements about reasonableness, including the conversion to appropriate units

MG1H-7 develops and carries out simple statistical processes to answer questions posed

MG1H-9 chooses and uses appropriate technology to organise information from a range of practical and everyday contexts

MG1H-10 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others.

Content summary

FSPe1CEC Water usage and collection

FSPe2CEC Electricity

FSPe3CEC Sustainability and energy-efficient housing

Terminology

appliance
area
availability
Building Sustainability Index (BASIX)
catchment
consumption
energy efficient
energy rating
grey water
interpretation
joule
kilolitre
kilowatt
power rating
resource
running cost
sustainability
volume
watt

Use of technology

Students should use spreadsheets to create graphs to display two or more sets of data, eg the rainfall in Cape York compared to the rainfall in Broken Hill.

Students could use spreadsheets or other appropriate software to calculate summary statistics.

The internet can be used to obtain suitable data for use in the study of topics within this Focus Study.

Notes

Water is an essential resource for life. In this Focus Study, students study aspects of water as a resource, including rainfall and personal water usage.

Electricity is a fundamental resource in modern life. In this Focus Study, students solve problems based on the consumption of household electricity.

Current information on the Building Sustainability Index (BASIX) Certificate is available on the internet.

FSPe1CEC Water usage and collection

In this topic, students interpret information, make comparisons, and perform a range of calculations in relation to personal water usage.

Outcomes addressed

MG1H-1, MG1H-2, MG1H-3, MG1H-4, MG1H-5, MG1H-7, MG1H-9, MG1H-10

Content

Students:

- interpret information about a household's water usage, eg a household water bill
- collect and interpret data and calculate statistics on household and personal water usage
- collect and interpret data and calculate statistics on local rainfall
- construct and interpret rainfall graphs
- calculate the volume of water collected, based on a catchment area, using $V = Ah$
- calculate the volume of water held by tanks of various shapes and sizes
- compare the amount of water used by a household and the amount of rainfall that could be collected over a given period
- calculate the amount of water that could be saved by initiating changes to household water use, including changing fittings, recycling grey water, and collecting and recycling stormwater.

Considerations

- Rainfall data is widely available on the internet. Useful information and data can be found at the Australian Bureau of Meteorology and Sydney Water websites.
- The cost of water may vary depending upon locality. Investigations of cost are not intended to be limited to town water only. Teachers should investigate relevant local methods of water accessibility and cost. Comparisons between the cost of water in cities and country areas should be made.
- Students should be aware that the millimetre is the standard unit of length on building plans.
- The catchment area of a roof is the 'plan view area' of the roof and not the actual area of the roofing material. It should be noted that the roof catchment area for single-storey houses is usually greater than the floor area if the house has eaves.
- There are some restrictions to the collection and uses of grey water. Students should be aware of safe grey water recycling practices, eg grey water should not be used to water edible garden plants and it should not be stored for longer than 24 hours.

Suggested applications

- Determine the amount and cost of water used by various household activities, including showering and bathing, washing clothes, watering the garden, washing a car, and using the toilet.
- Students log their water use over a one-week period and use this to estimate personal water usage and costs over longer time periods.
- Students use the following rule-of-thumb approach to estimating the collection capacity of a roof:

  Every 1 mm of rain = 1 litre (L) of water per square metre (m^2) of roof area, and then allow for a 15% wastage factor.
- From a plan of a house, students determine and justify the best location for a rainwater tank, taking into account the collection area of different sections of the roof and where the recycled water is to be used.
- Students investigate the possible uses of collected stormwater within the household.
- Students create area charts of the amount of stormwater collected versus the amount of water used by the household for watering lawns and gardens, flushing toilets, and running washing machines.
- Perform calculations to compare the amount of water used by products with various ratings, eg dishwashers and washing machines.

FSPe2CEC Electricity

The principal focus of this topic is the calculation and comparison of household electricity consumption and cost, and the calculation and interpretation of related statistics.

Outcomes addressed

MG1H-3, MG1H-5, MG1H-9, MG1H-10

Content

Students:

- interpret information about a household's electricity usage, eg a household electricity bill
- calculate the cost of running different household appliances for various time periods, given the power rating, usage time, and cost of power
- interpret the energy rating of appliances and compare running costs of different models of the same type of appliance
- calculate the amount of electricity that could be saved by using energy-efficient devices and adopting energy-efficient strategies.

Considerations

- The watt is the International System of Units (SI) derived unit of power and is equal to one joule per second. By definition power is a rate. The symbol for the watt is W.
- Students should be aware of energy rating labels and be able to interpret the symbols and figures.

Suggested applications

- Calculations of running costs of various appliances, eg
  - Calculate the cost of running a 200-watt television for six hours if the average peak rate for domestic electricity is $\$0.15$/kWh.

    Solution

    Total electricity used = 200×6 = 1200 watt-hours or 1.2 kWh.

    Cost of electricity used is 1.2 kWh $\times$ $\$0.15$/kWh = $\$0.18$.
  - Calculate the cost of running a 2400-watt (2.4 kW) fan heater for eight hours per day for 30 days. Assume electricity is charged at $\$0.18$/kWh.

    Solution

    Total electricity used = $2.4 \times 8 \times 30$ = 576 kWh.

    Cost of electricity used is 576 kWh $\times$ $\$0.18$/kWh = $\$103.68$.
- Students investigate the energy rating of different appliances and make relevant calculations of the cost to run those appliances for a given period of time. A comparison of the cost of running various appliances with different energy ratings could be made.

FSPe3CEC Sustainability and energy-efficient housing

In this topic, students interpret information, make comparisons, and perform a range of calculations in relation to requirements and data relevant to sustainability and energy-efficient housing.

Outcomes addressed

MG1H-3, MG1H-5, MG1H-7

Content

Students:

- calculate building sustainability measures based on the requirements of the Building Sustainability Index (BASIX) Certificate
- identify the issues addressed in the BASIX, eg area of site, water, thermal comfort, and energy
- calculate the site area of a proposed development, given a site plan
- calculate the roof area of a building from a plan
- calculate the ratio of floor area for which air conditioning applies to the floor area for which air conditioning does not apply
- calculate garden and lawn area, including low and high water-use areas
- calculate the volume of a tank, swimming pool and/or spa
- calculate the floor area of a building from a plan
- interpret measurements of wall heights and wall thicknesses from a plan
- measure the size of eaves from a plan
- calculate the amount of roof insulation required for a building from a plan
- determine the orientation of windows and skylights using a plan
- determine the breeze path on a plan.

Considerations

- All building plan dimensions are in millimetres.
- Teachers may wish to restrict the design of swimming pools and spas for ease of calculation, or discuss and investigate the implications of irregular-shaped pools and pools of varying depth.
- Local information should be discussed wherever possible, eg local climate and rainfall, and types of native vegetation.

Suggested applications

- Make calculations for a BASIX checklist, including all area and volume calculations, and determine the orientation of the dwelling, windows and skylights, from a site plan of a proposed dwelling.
- Students use a compass to assist in the determination of orientation of a dwelling or the windows, doors and skylights, eg see the compass/sector diagram from the BASIX website.
- Design an energy-efficient house for a particular building site. The plan should consider as many aspects of energy efficiency as possible, eg which direction the house faces to minimise cooling and heating costs, the inclusion of solar power, ceiling fans, insulation methods, and rainwater collection.

15 HSC Mathematics General 1: Assessment and Reporting

15.1 Requirements and Advice

The information in this section of the syllabus relates to Board of Studies requirements for assessing student achievement in the Content Endorsed Courses for the Higher School Certificate.

Assessment is the process of gathering information and making judgements about student achievement for a variety of purposes. Those purposes include:

- assisting student learning
- evaluating and improving teaching and learning programs
- providing evidence of satisfactory achievement and completion in the Preliminary Mathematics General course
- providing the Higher School Certificate results.

15.2 Assessment of Stage 6 Content Endorsed Courses

There is no external examination of students in Stage 6 Content Endorsed Courses.

Assessment provides a measure of a student's achievement based on the range of syllabus content and outcomes. The assessment components, weightings and task requirements to be applied to internal assessment are identified in section 15.3. They ensure a common focus across schools for internal assessment in the course, while allowing for flexibility in the design of tasks. A variety of tasks should be used to give students the opportunity to demonstrate outcomes in different ways and to improve the validity and reliability of the assessment.

Schools should develop an assessment program that:

- specifies the various assessment tasks and the weightings allocated to each task
- provides a schedule of the tasks designed for the whole course.

The school should also develop and implement procedures to:

- inform students in writing of the assessment requirements for each course before the commencement of the HSC course
- ensure that students are given adequate written notice of the nature and timing of assessment tasks
- provide meaningful feedback on each student's performance in all assessment tasks
- maintain records of marks awarded to each student for all assessment tasks
- address issues relating to illness, misadventure and malpractice in assessment tasks
- address issues relating to late submission and non-completion of assessment tasks
- advise students in writing if they are not meeting the assessment requirements in a course and indicate what is necessary to enable the students to satisfy the requirements
- inform students about their entitlements to school reviews and appeals to the Board
- conduct school reviews of assessments when requested by students.

15.3 Assessment Components, Weightings and Tasks

The components and weightings to be used by schools are detailed below. The allocation of weighting to particular tasks is left to the individual schools, but the percentage allocated to each assessment component must be maintained.

There should be a balance between the assessment of:

- knowledge and understanding outcomes and course content

  and

- skills outcomes and content as follows:

Component	Weighting (%)
Knowledge and understanding	50
Skills	50

One task may be used to assesses several components. It is suggested that three to five tasks are sufficient to assess the HSC course outcomes for a two-unit course.

The assessment tasks given to students must:

- be consistent with the type of objectives and outcomes being assessed
- provide for a range of performances and achievements within the group
- be consistent in number with comparable 2 unit Board Developed Courses
- use a range of assessment instruments – each instrument must be appropriate to the outcomes it is designed to measure.

Other requirements:

- At least one assessment task must derive from formal examinations. Formal examinations are defined as any form of examination as used in the Higher School Certificate under conditions similar to those in the HSC for comparable tasks and that apply equally to all students at the school.

Instruments used for assessment purposes may include the following:

- short-answer tests
- essay tests
- practical demonstrations
- portfolios of useful resources
- problem-solving assignments, eg design and make
- interviews/talks
- diary/learning logs
- internet research assignments
- critical reviews
- physical products
- observation reports
- stimulus questions
- multiple-choice tests
- research projects
- written reports on case studies, excursions, field trips, surveys and investigations
- lesson plans
- role plays
- debates
- oral reports
- individual/group reports
- mock interviews

The keeping of an outcomes diary in which students evaluate their learning could be a useful portfolio inclusion.

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16 Post-school Opportunities

The Preliminary Mathematics General/HSC Mathematics General 2 pathway provides a strong foundation for a broad range of vocational pathways, as well as for a range of university courses. The Preliminary Mathematics General/HSC Mathematics General 1 pathway provides an appropriate foundation for a range of such vocational pathways, either in the workforce or in further training.

In addition, the study of the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course assists students in preparing for employment and full and active participation as citizens. In particular, there are opportunities for students to gain recognition in vocational education and training. Teachers and students should be aware of these opportunities.

Recognition of Student Achievement in Vocational Education and Training (VET)

Students may have the knowledge and skills that they have developed through the study of HSC courses recognised by industry and training organisations. Students can claim recognition towards courses or qualifications if they are able to demonstrate that they have already achieved some of the course outcomes. The effect of such recognition is that the student is exempted from certain course requirements and will not have to repeat their learning in courses with TAFE NSW or other Registered Training Organisations (RTOs).

The degree of recognition available to students for each HSC subject is based on the similarity of outcomes between HSC courses and the qualification(s) and/or unit(s) of competency from industry training packages. More information about industry training packages can be found on the training.gov.au website.

Recognition by TAFE NSW

TAFE NSW conducts courses in a wide range of industry areas. Information about the range of courses 7available can be found on the TAFE NSW website (http://www.tafensw.edu.au). Under current arrangements, the recognition available to students of mathematics in relevant courses conducted by TAFE is outlined on the HSC/TAFE Credit Transfer website (https://www.det.nsw.edu.au/). Teachers should refer to this website and be aware of the recognition that may be available to their students through the study of the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course.

Recognition by other Registered Training Organisations

Under the requirements of the Australian Quality Training Framework (AQTF) 2007, all RTOs are required to offer recognition of prior learning for training package qualifications and/or units of competency. A student can apply to an RTO for recognition of knowledge and skills gained in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course. The student would need to provide the RTO with evidence of satisfactory achievement in the Preliminary Mathematics General course, the HSC Mathematics General 2 course or the HSC Mathematics General 1 course so that the degree of recognition available can be determined.