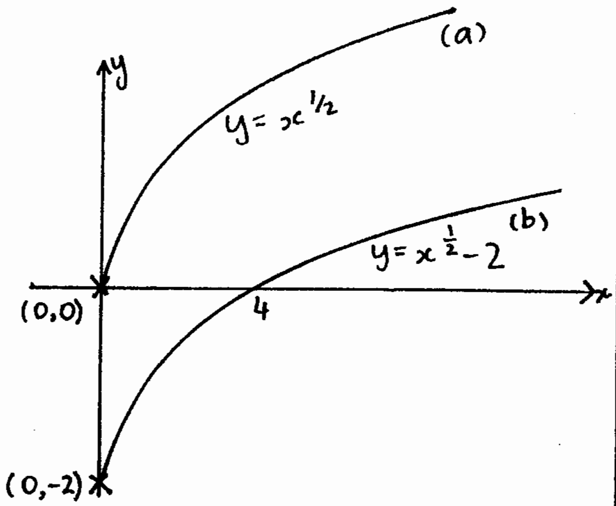
GRAPHS — D & G - ARNOLD

EXERCISE: 1.1

1.



(a) $\dfrac{dy}{dx} = \dfrac{1}{2} x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

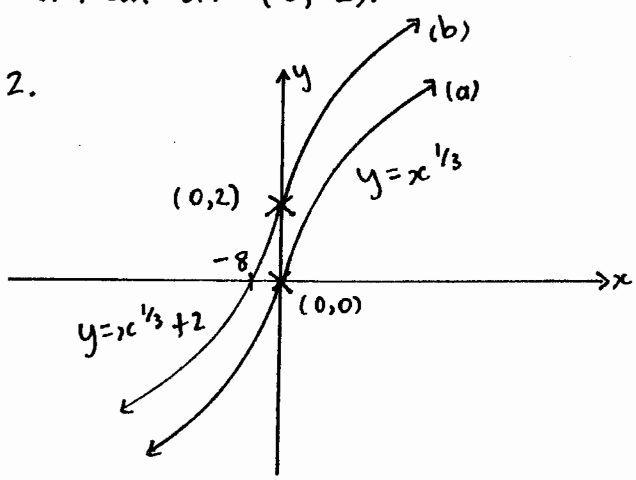
when $x \to 0^+$, $\dfrac{dy}{dx} \to +\infty$

$\therefore \dfrac{dy}{dx}$ is not defined.

$\therefore$ Tangent at $(0, 0)$ is vertical.

(b) Similarly, the tangent is vertical at $(0,-2)$.

2.



(a) $\dfrac{dy}{dx} = \dfrac{1}{3} x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$

when $x \to 0^+$, $\dfrac{dy}{dx} \to +\infty$

when $x \to 0^-$, $\dfrac{dy}{dx} \to -\infty$

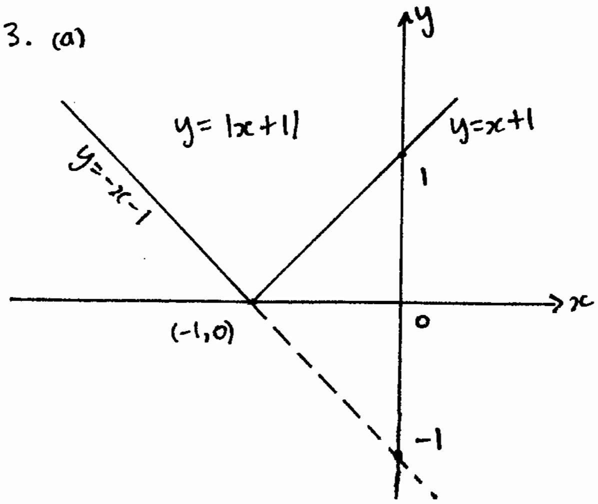
$\therefore \dfrac{dy}{dx}$ is not defined at $(0,0)$.

$\therefore$ Tangent at $(0,0)$ is vertical, also The tangent at $(0,2)$ is vertical.

[$(0,2)$ is a critical point]

(b) Similarly, the tangent is vertical at $(0,0)$ [critical point]

3. (a)

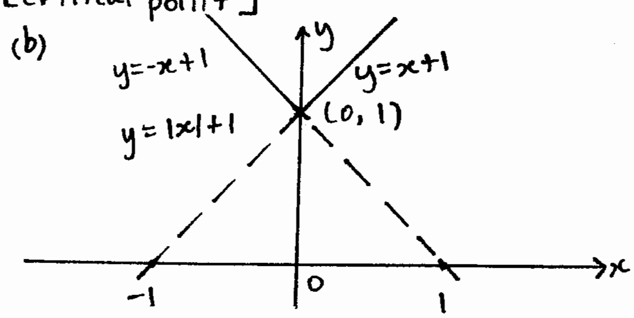


$\dfrac{dy}{dx} = -1$ when $x \to -1^-$

$\dfrac{dy}{dx} = 1$ when $x \to -1^+$

$(-1,0)$ is an angular point [critical point]

(b)



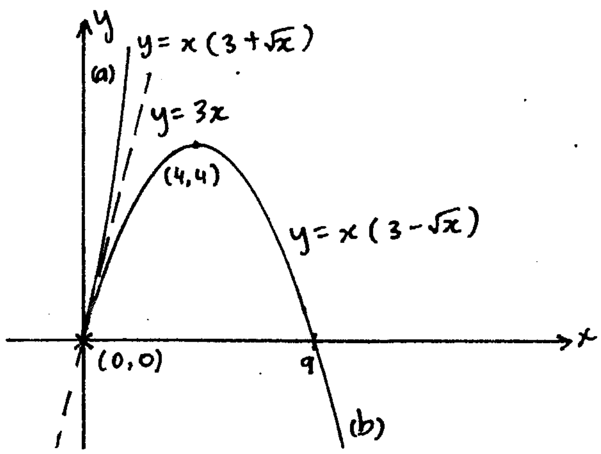
$\dfrac{dy}{dx} = -1$ when $x \to 0^-$

$\dfrac{dy}{dx} = 1$ when $x \to 0^+$

$(0,1)$ is an angular point

$\therefore (0,1)$ is a critical point, where $\dfrac{dy}{dx}$ is not defined.

4.



(a) $\frac{dy}{dx} = 3 + \sqrt{x} + \left(\frac{1}{2}x^{-\frac{1}{2}}\right)x$

$= 3 + \sqrt{x} + \frac{1}{2\sqrt{x}} \times x$

$= 3 + \sqrt{x} + \frac{x}{2\sqrt{x}} = \frac{6\sqrt{x} + 3x}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{6\sqrt{x} + 3x}{2\sqrt{x}}$ when $x \to 0^+$

$\therefore \frac{dy}{dx}$ is not defined.

$\therefore$ Tangent at $(0,0)$ is vertical.

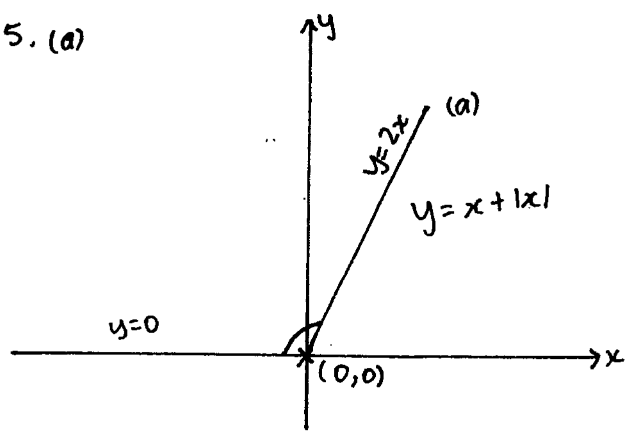
(b) $\frac{dy}{dx} = 3 - \sqrt{x} + \left(-\frac{1}{2\sqrt{x}}\right) \times x$

$= 3 - \sqrt{x} - \frac{x}{2\sqrt{x}} = \frac{6\sqrt{x} - 3x}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{6\sqrt{x} - 3x}{2\sqrt{x}}$ when $x \to 0^+$

$\therefore$ Tangent at $(0,0)$ is vertical.

5. (a)

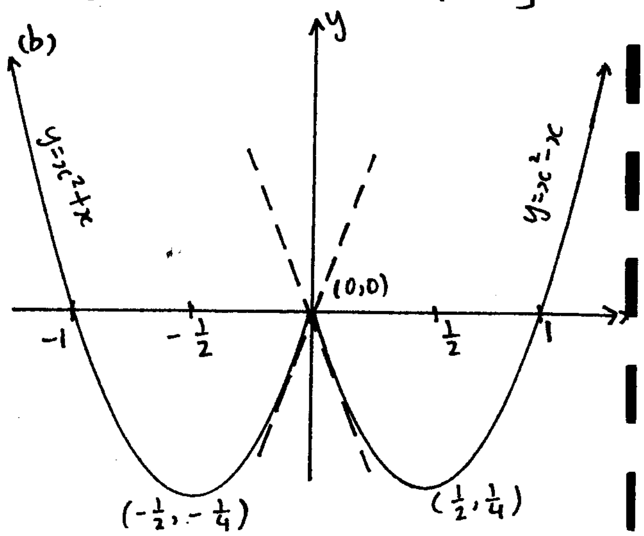


$\frac{dy}{dx} = 2$ when $x \to 0^+$

$\frac{dy}{dx} = 0$ when $x \to 0^-$

$\therefore (0,0)$ is an angular point.
[(0,0) is a critical point]

(b)

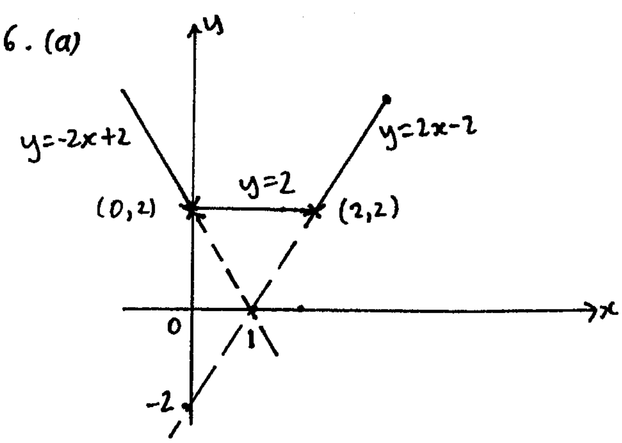


$\frac{dy}{dx} = 2x + 1$ when $x \to 0^-$

$\frac{dy}{dx} = 2x - 1$ when $x \to 0^+$

$\therefore$ Tangent at $(0,0)$ is vertical.

6. (a)

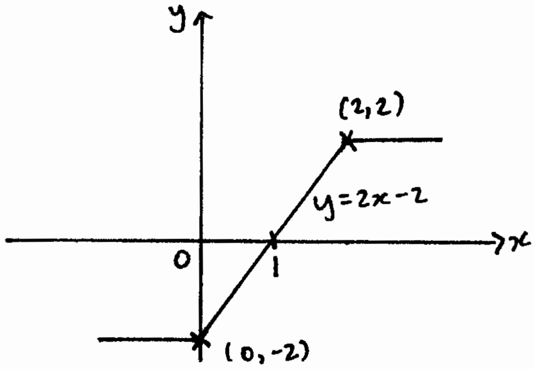


$\frac{dy}{dx} = -2$ when $x \to 0^-$

$\frac{dy}{dx} = 0$ when $x \to 0^+$

The gradient at 0^- is -2.
The gradient at 0^+ is 0.

(b)



$\frac{dy}{dx} = 0$ when $x \to 2^+$

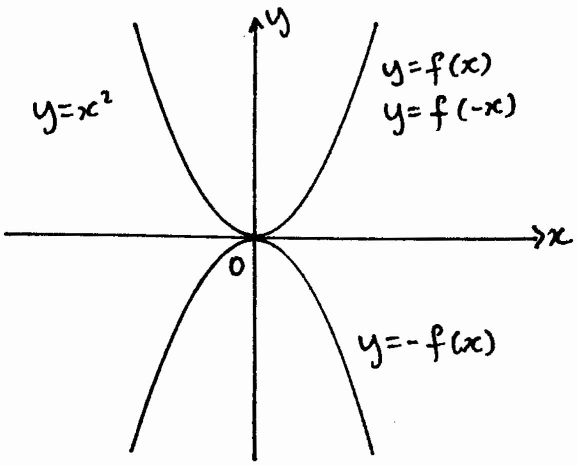
$\frac{dy}{dx} = 2$ when $x \to 0^+$

$\frac{dy}{dx} = 0$ when $x \to 2^-$

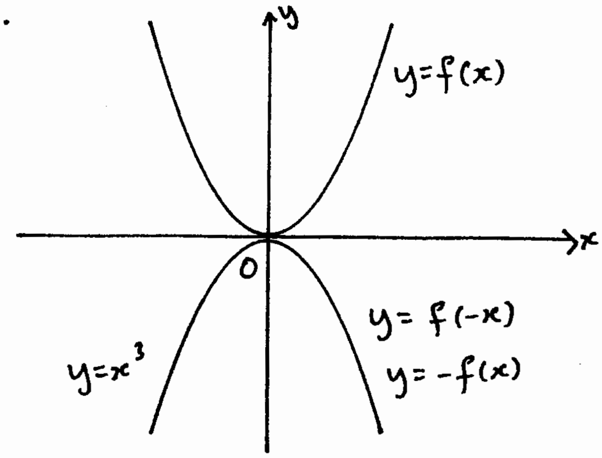
At the points $(0,-2)$ and $(2,2)$, $\frac{dy}{dx}$ is not defined since at each of them the derivative to the left is not equal to the derivative to the right. These 2 critical points can be also called angular points.

EXERCISE : 1·2

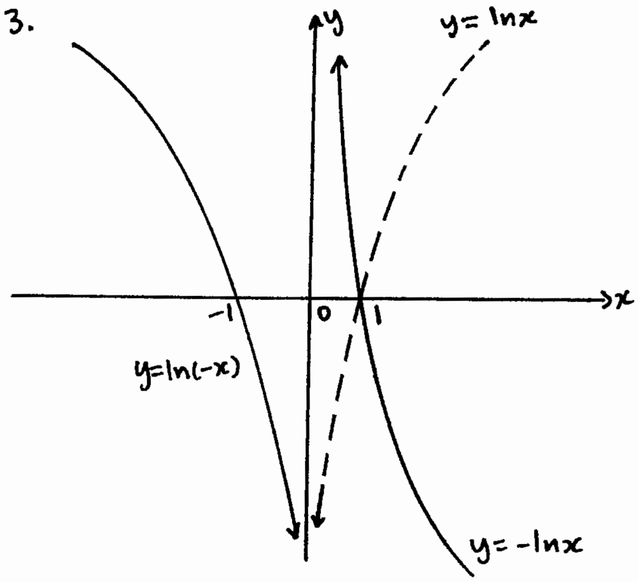
1.



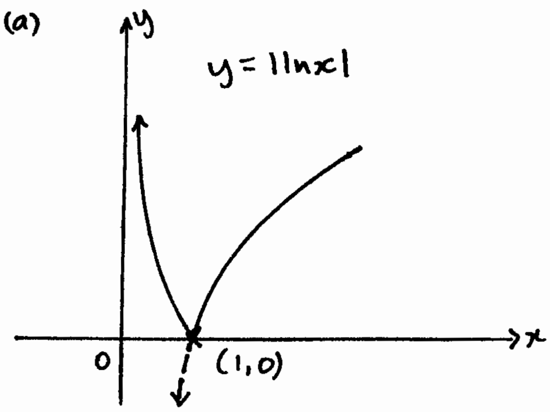
2.



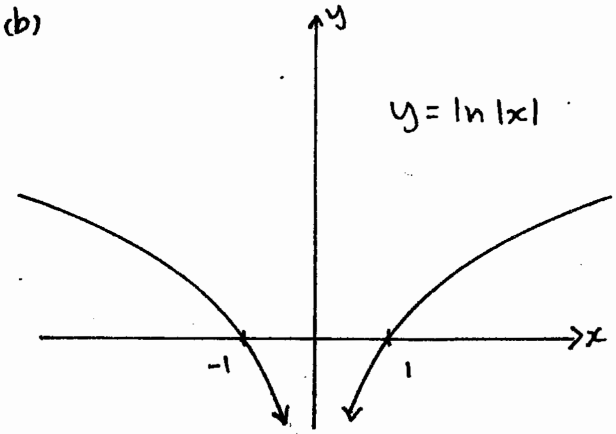
3.



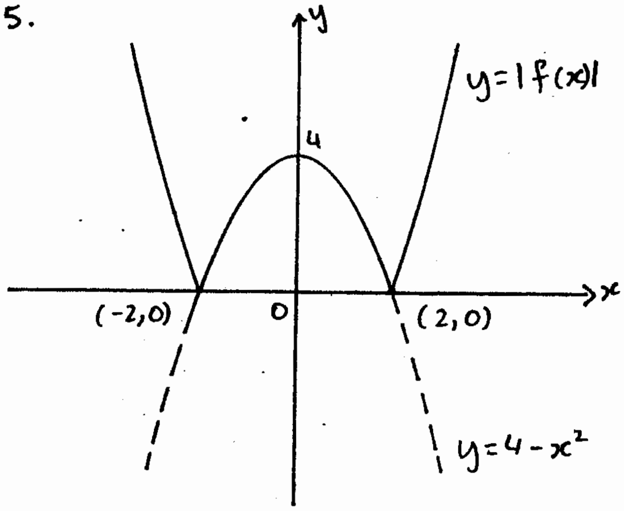
4. (a)



(b)

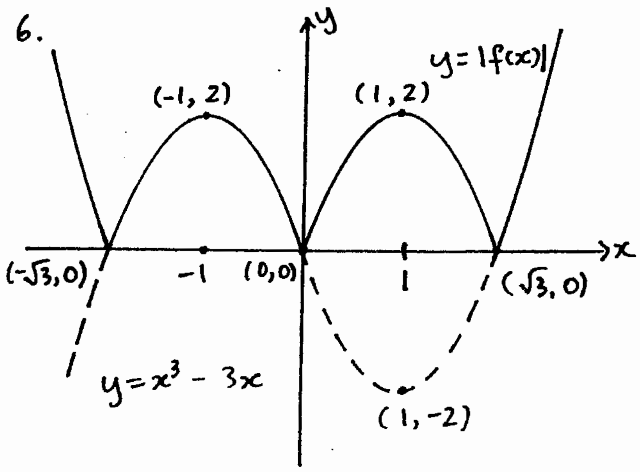


5.



∴ The graph is an even function as it is symmetrical about the y-axis and/or $f(-x) = f(x)$

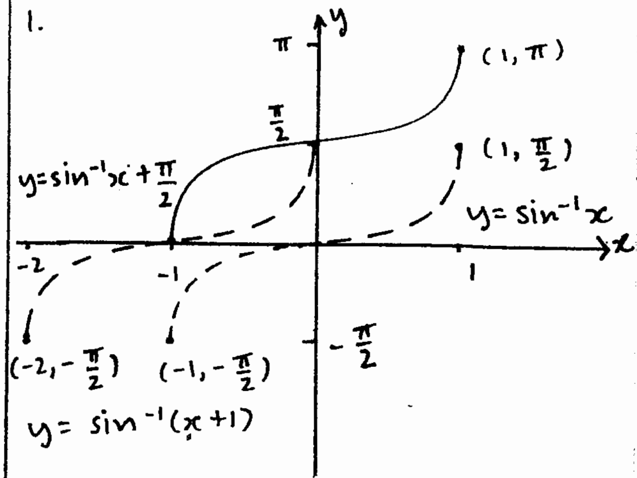
6.



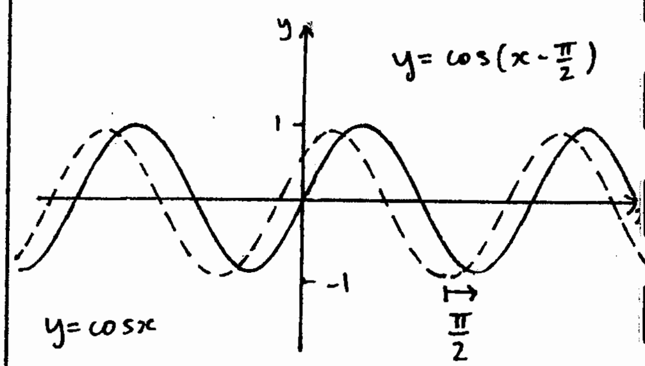
∴ The graph is an even function as it is symmetrical about the y-axis and/or $f(-x) = f(x)$

EXERCISE: 1.3

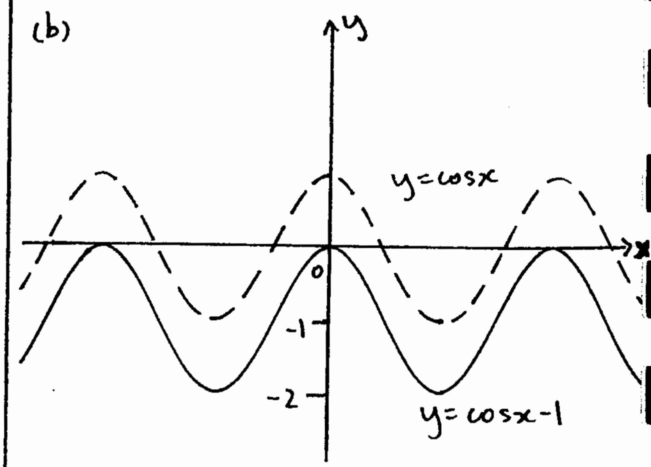
1.



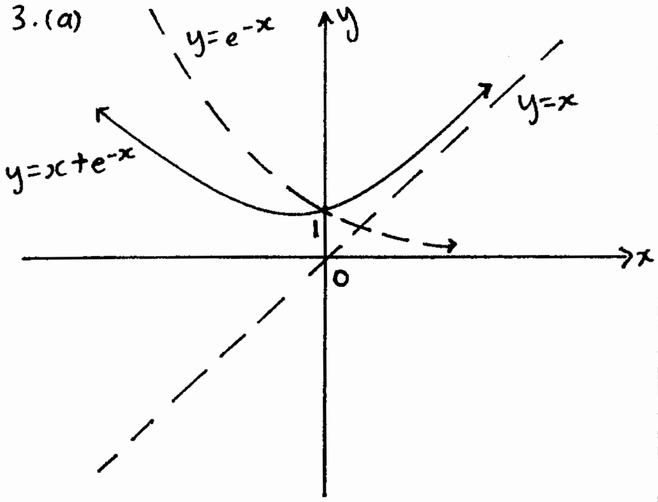
2. (a)



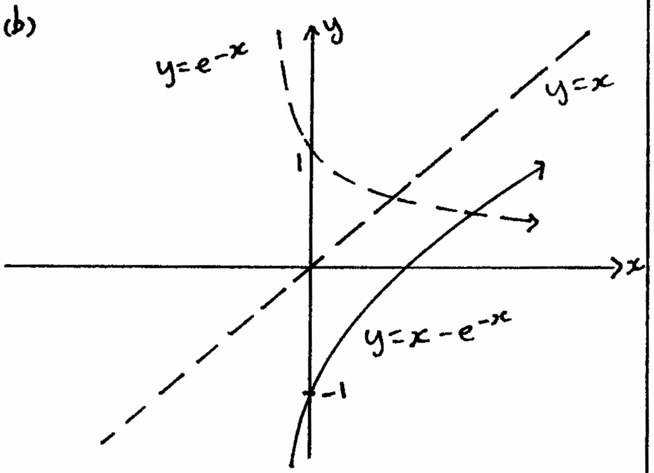
(b)



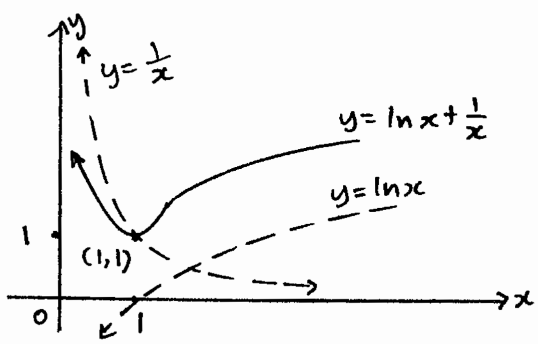
3. (a)



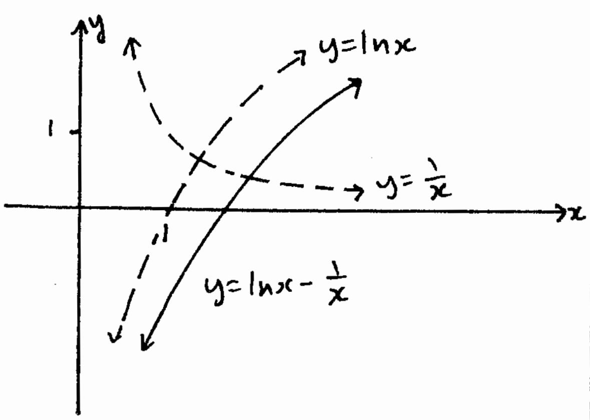
(b)



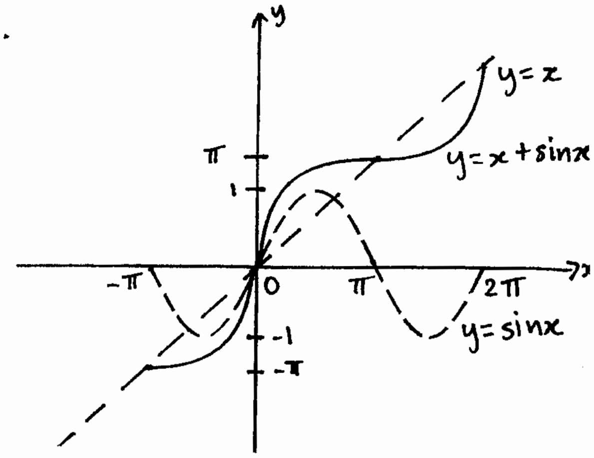
4. (a)



(b)



5.



Yes, it is an odd function because it is symmetrical about the origin. However, we can do it mathematically:

$f(x) = x+\sin x$

$\therefore f(-x) = -x+\sin(-x) = -x-\sin x$

$\therefore f(-x) = -f(x)$

$\therefore f(x)$ is an odd function.

6. As $f(x) = g(x) + h(x)$

$\therefore f(-x) = g(-x) + h(-x)$

but $g(x) = g(-x)$ [$g(x)$ is an even function]

$h(x) = h(-x)$ [$h(x)$ is an even function]

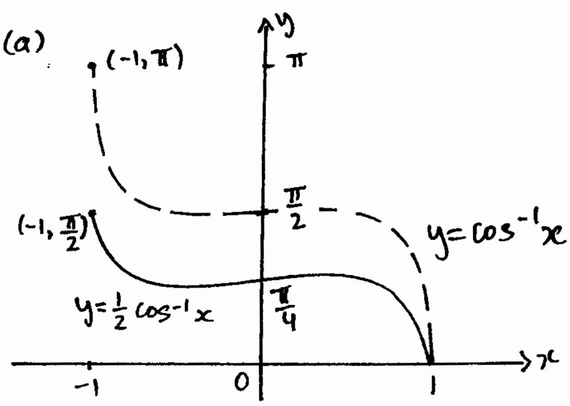
$\therefore f(-x) = g(x) + h(x)$

$\therefore f(x) = f(-x)$

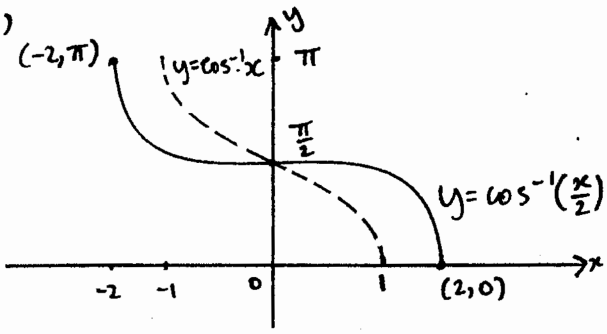
$\therefore f(x)$ is an even function.

EXERCISE: 1.4

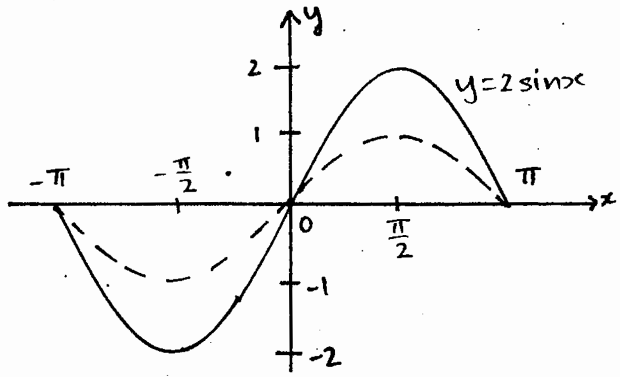
1. (a)



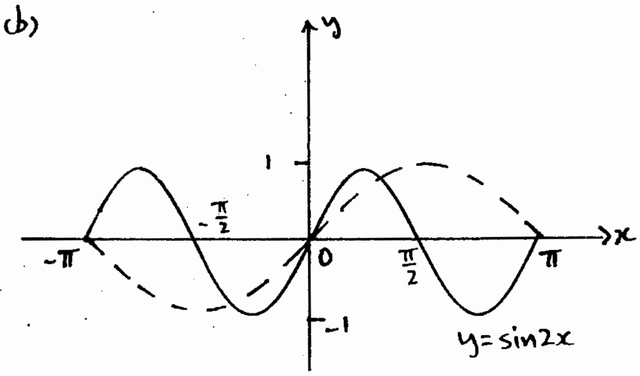
(b)



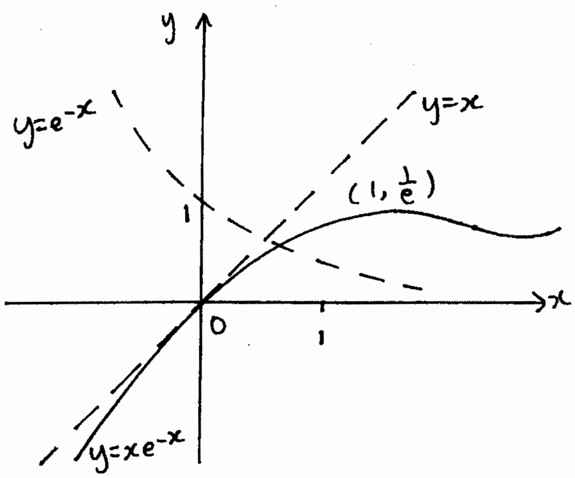
2. (a)



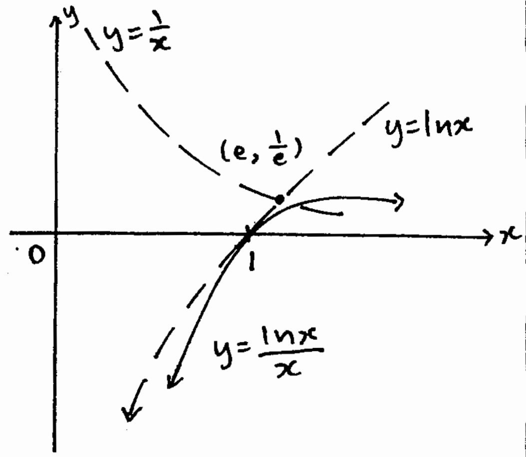
(b)



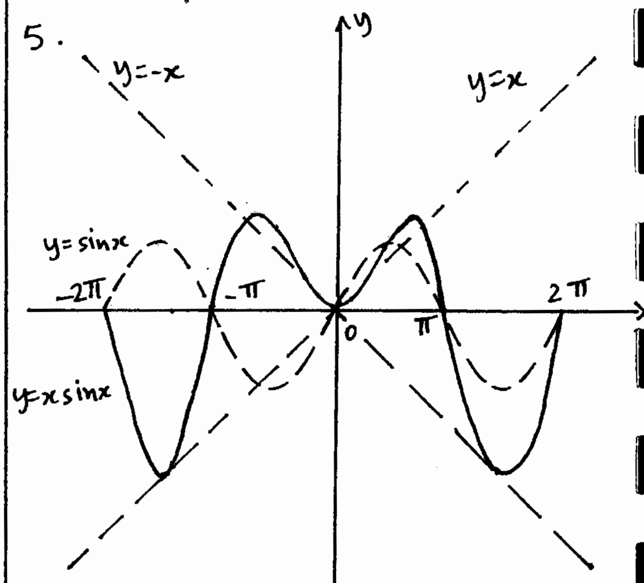
3.



4.



5.



Yes, it is an odd function since it is symmetrical about the origin (0, 0). We can prove this also as

$f(x) = x \sin x$

$\therefore f(-x) = -x \sin(-x) = x \sin x$

$\therefore f(-x) = f(x)$

$\therefore f(x)$ is an even function.

6. As $f(x) = g(x)\, h(x)$

$\therefore f(-x) = g(-x)\, h(-x)$

but $g(-x) = g(x)$ and

$h(-x) = h(x)$

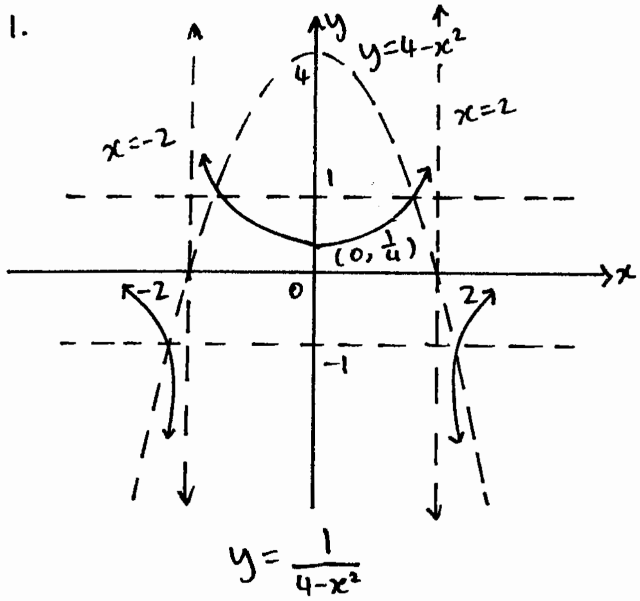
[since both $g(x)$ and $h(x)$ are even]

$\therefore f(-x) = g(x)\, h(x)$ $\therefore f(-x) = f(x)$

$\therefore f(x)$ is an even function.

EXERCISE: 1·5

1.

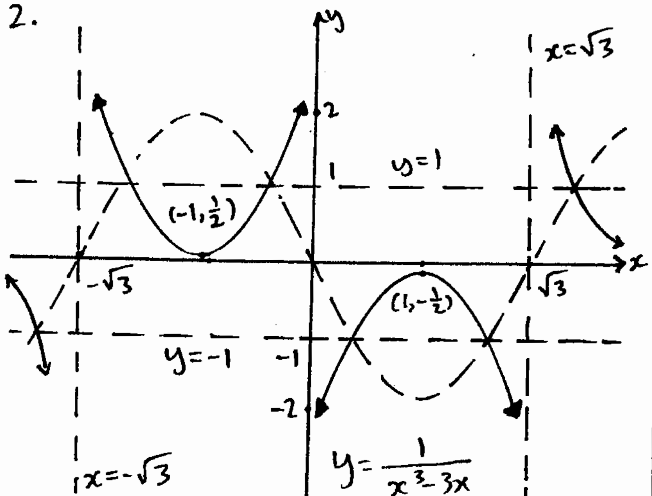


Yes, because it's symmetrical about the y-axis. $g(x) = \frac{1}{4-x^2}$

$\therefore g(-x) = \frac{1}{4-(-x)^2} = \frac{1}{4-x^2} = g(x)$

$\therefore g(x)$ is an even function.

2.



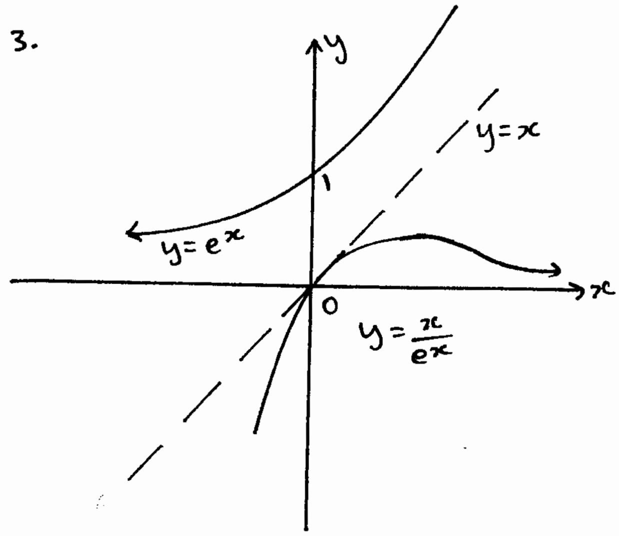
Note: When taking reciprocal of a function f(x) every point of intersection with x-axis becomes an asymptote for $y = \frac{1}{f(x)}$.

As $g(x) = \frac{1}{x^3-3x}$

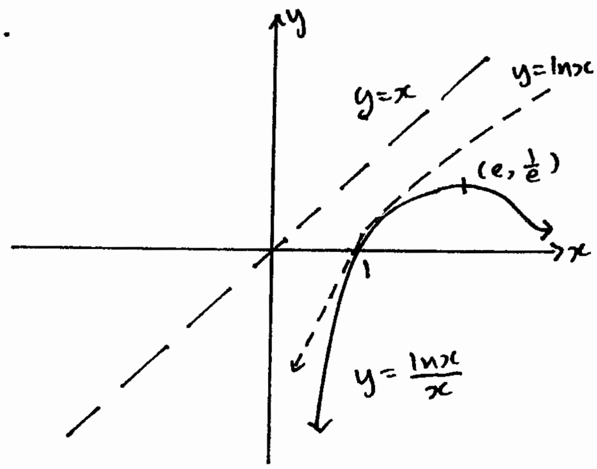
$\therefore g(-x) = \frac{1}{(-x)^3 - 3(-x)} = \frac{1}{-x^3+3x} = -g(x)$

$\therefore g(x)$ is an odd function and is symmetrical about origin as shown.

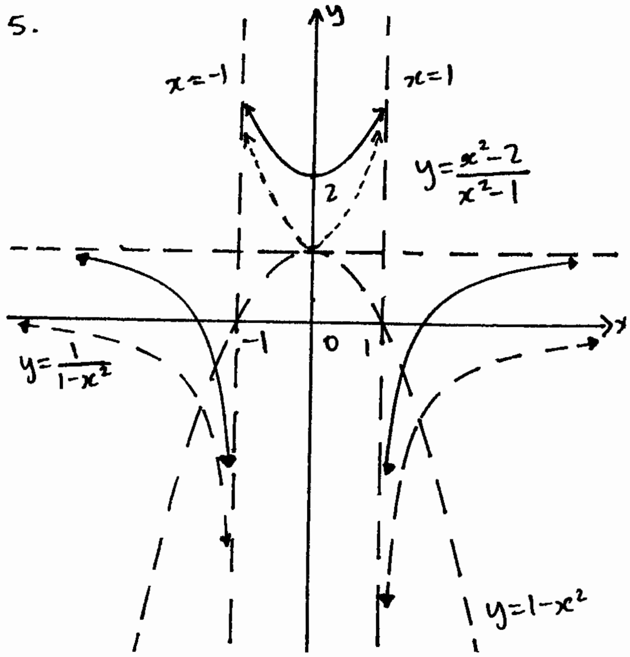
3.



4.



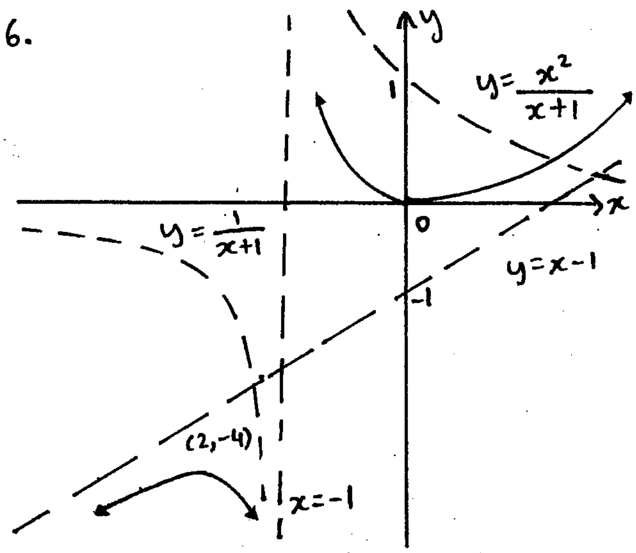
5.



$$y=\frac{x^2-2}{x^2-1}=\frac{x^2-1-1}{x^2-1}=1-\frac{1}{x^2-1}$$

$$\therefore y=1+\frac{1}{1-x^2}$$

6.



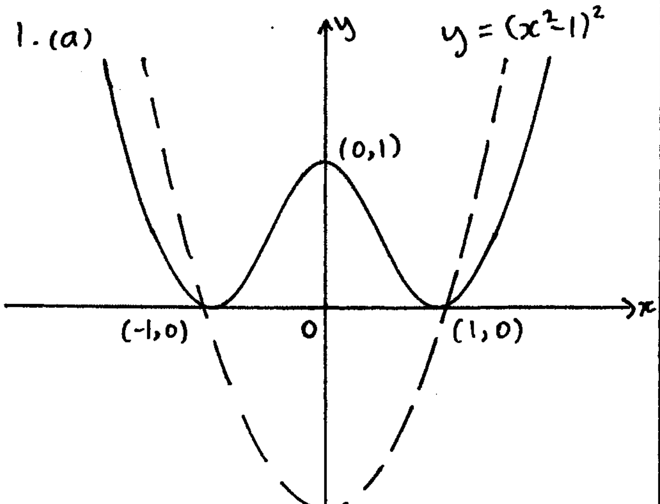
$$x+1\,\overline{)\,x^2\,}\quad \text{quotient } x-1$$

$$\begin{array}{r} x^2+x \\ \hline -x \\ -x-1 \\ \hline 1 \end{array}$$

$$\therefore y=\frac{x^2}{x+1}=x-1+\frac{1}{x+1}$$

EXERCISE: 1·6

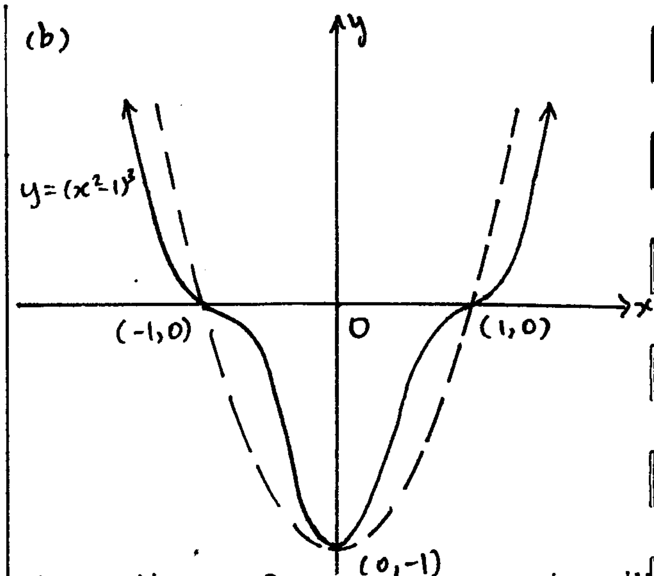
1. (a)



note:

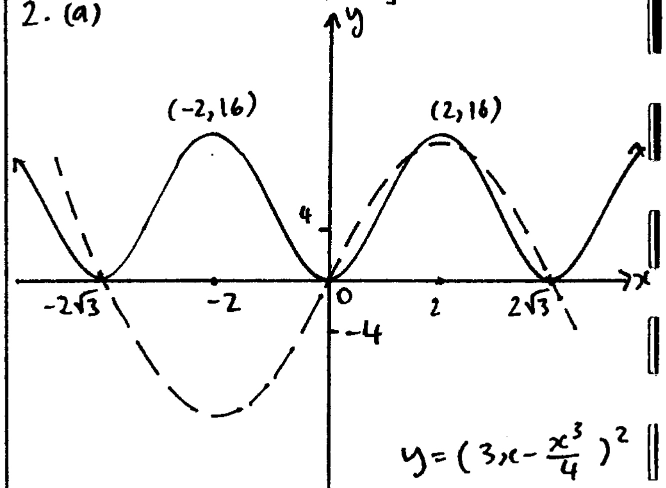
When squaring a function $f(x)$, every intersection with x-axis becomes a minimum turning point for $y=[f(x)]^2$

(b)

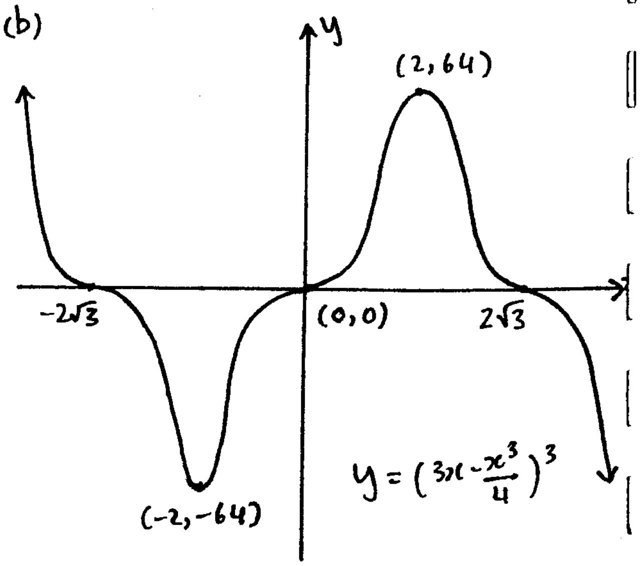


when cubing $y=f(x)$, every intersection with y-axis becomes a horizontal point of inflexion for $y=[f(x)]^3$.

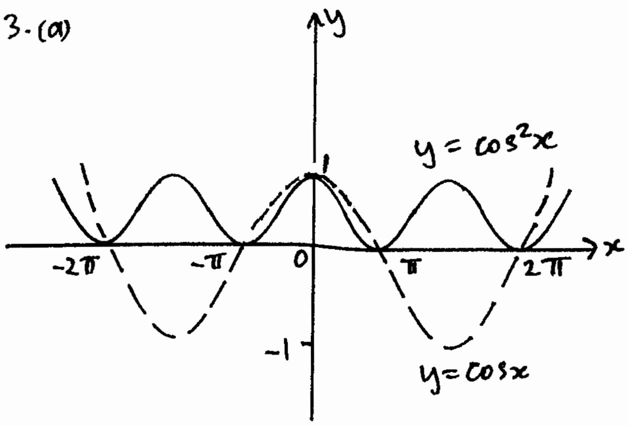
2. (a)



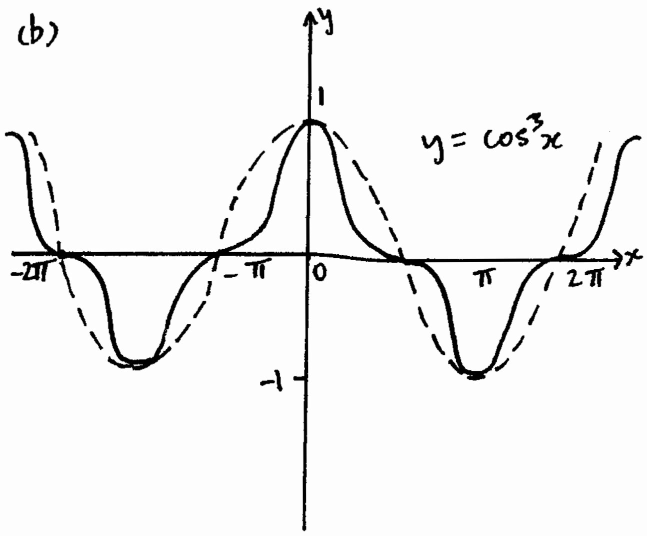
(b)



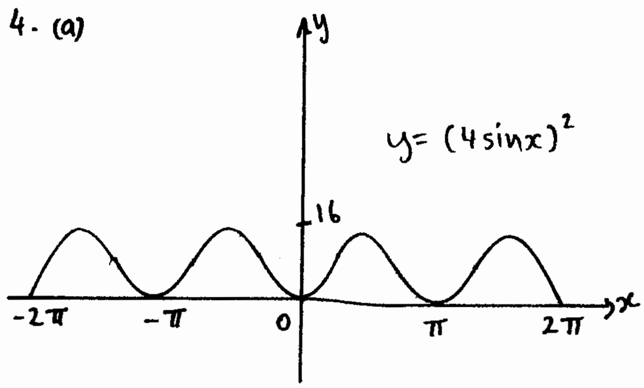
3. (a)



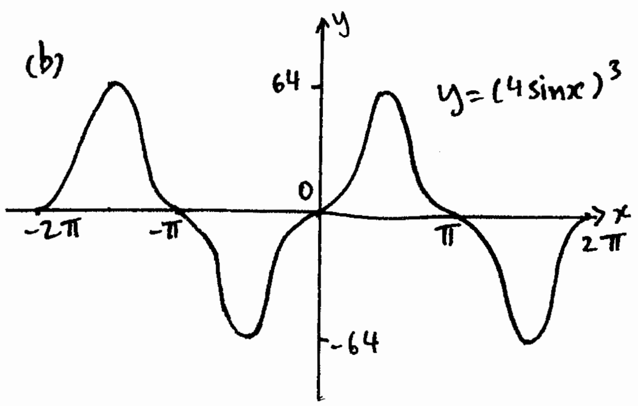
(b)



4. (a)

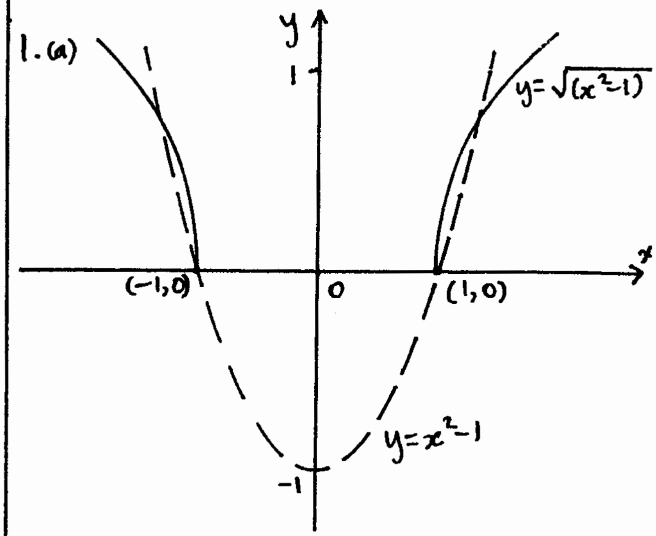


(b)

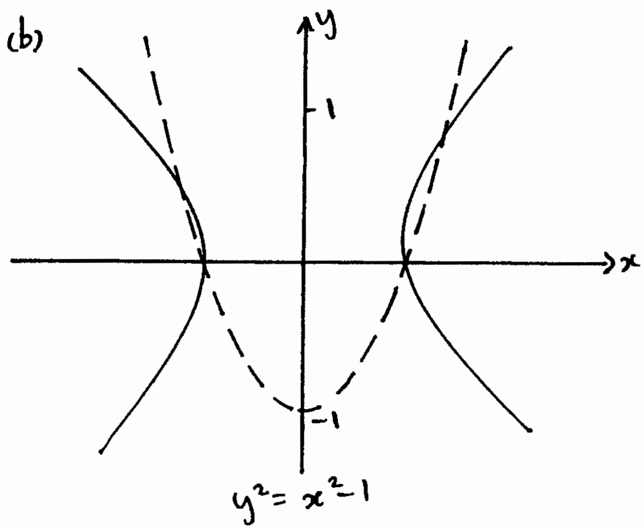


EXERCISE: 1·7

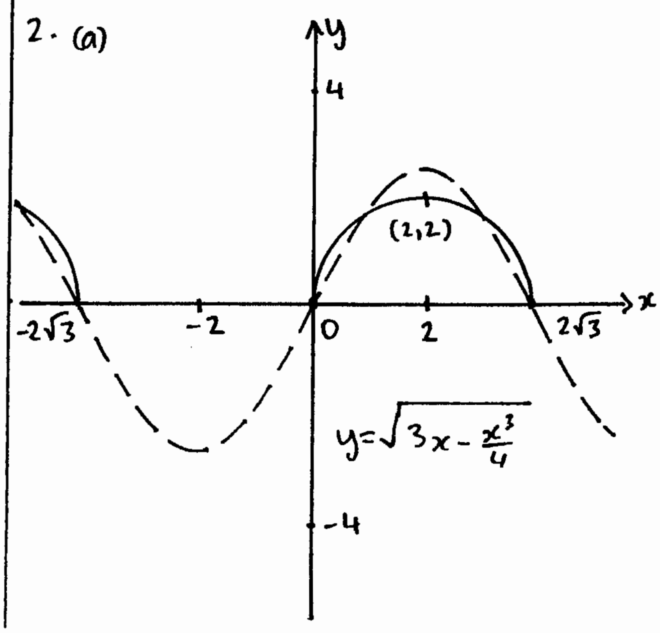
1. (a)



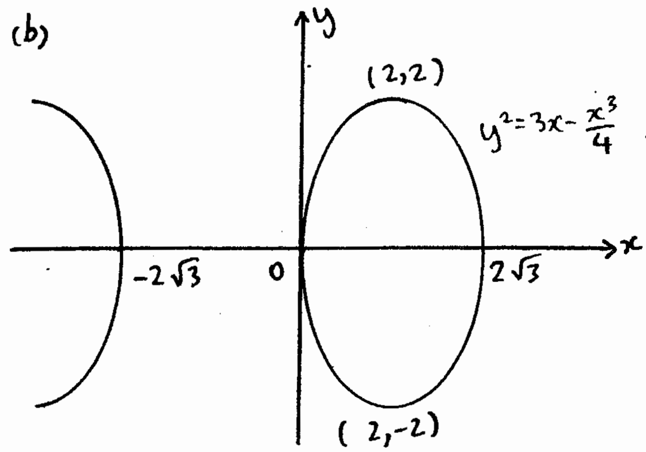
(b)



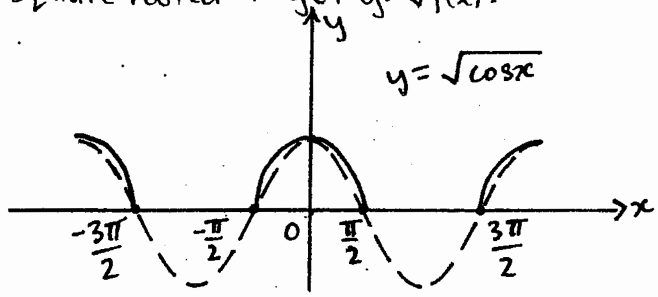
2. (a)



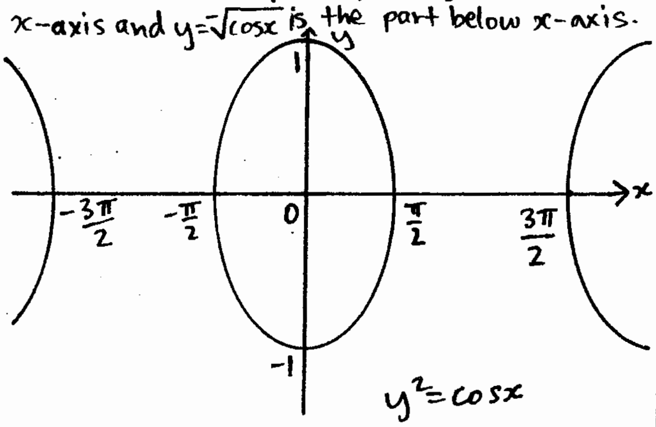
(b)



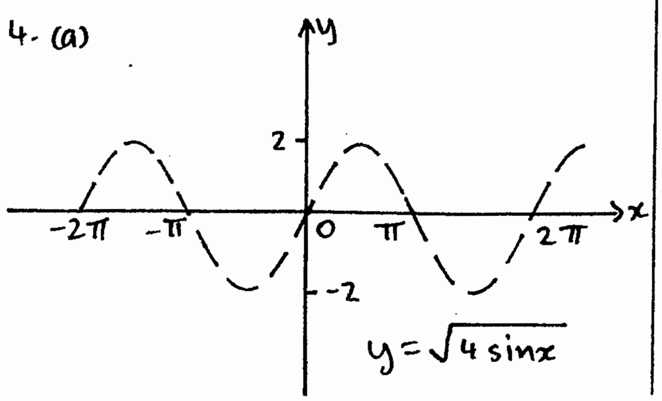
3.(a) When square rooting $y=f(x)$ every value of y less than 0 will nolonger exist and every positive value of y will be square rooted to get $y=\sqrt{f(x)}$.



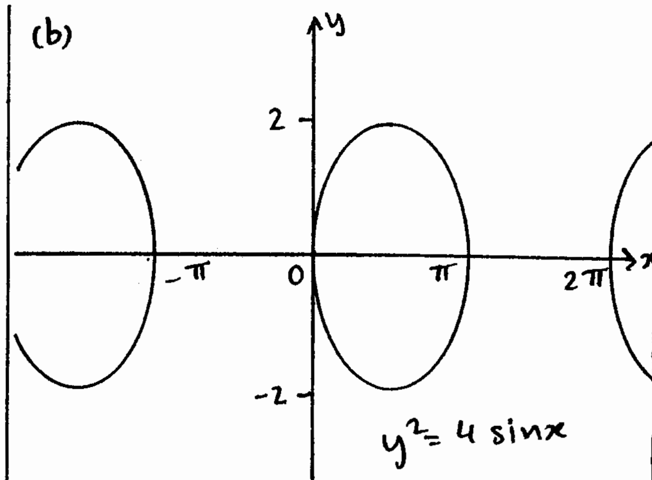
(b) $y^2=\cos x$ $\therefore y=\pm\sqrt{\cos x}$ where $y=\sqrt{\cos x}$ is the part of the graph above x-axis and $y=-\sqrt{\cos x}$ is the part below x-axis.



4. (a)

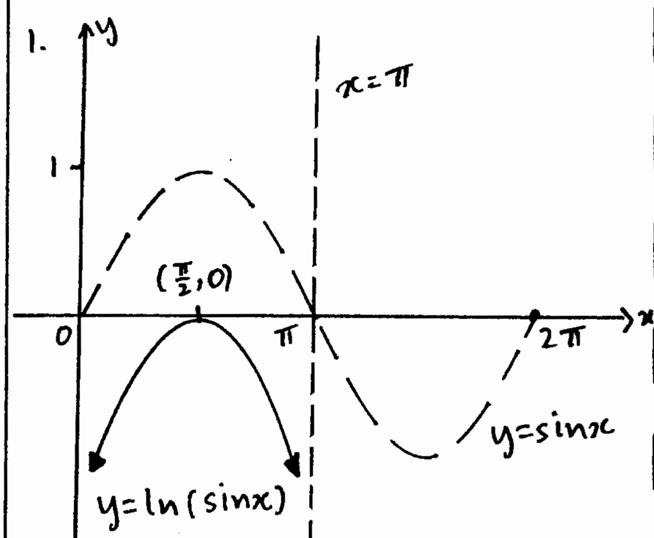


(b)

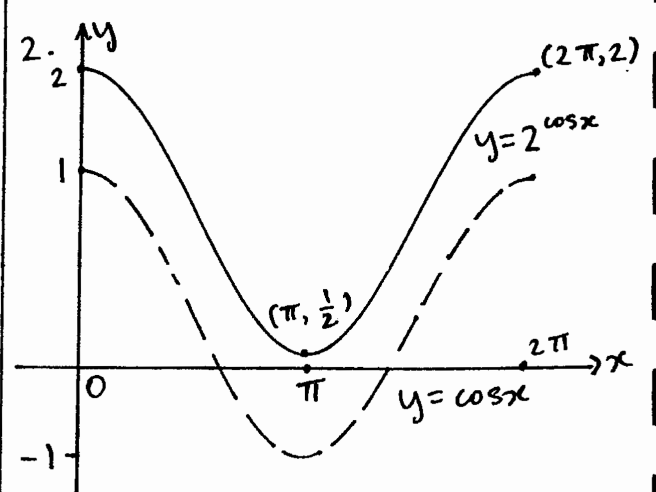


EXERCISE : 1.8

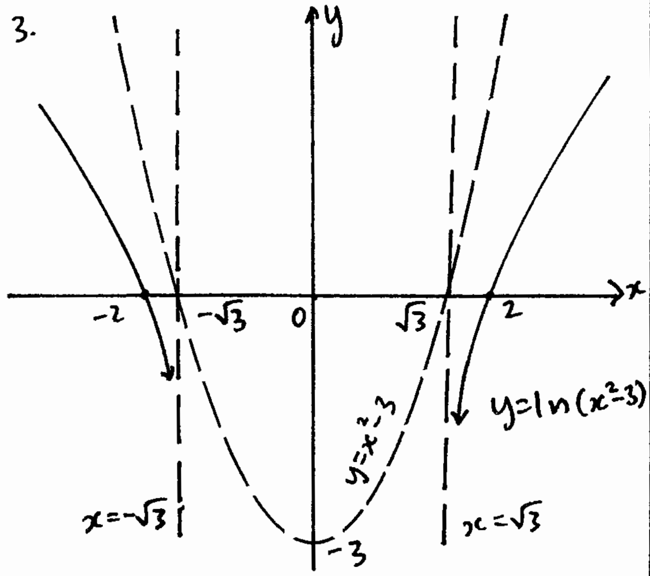
1.



2.



3.



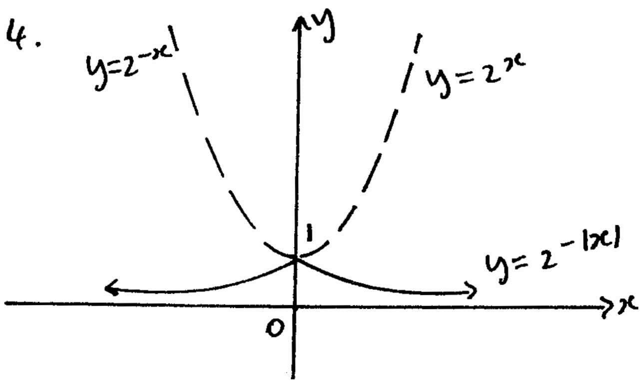
$f(x) = \ln(x^2-3)$

$\therefore f(-x) = \ln[(-x)^2-3] = \ln(x^2-3)$

$\therefore f(-x) = f(x)$

$\therefore f(x)$ is an even function. Also, we can see that it is symmetrical on the y-axis to further verify this property.

4.



$f(x) = 2^{-|x|}$

$\therefore f(-x) = 2^{-|-x|} = 2^{-|x|}$ (since $|-x| = |x|$)

$\therefore f(x) = f(-x)$

$\therefore f(x)$ is an even function. Also, we can see that it is symmetrical on the y-axis to further verify this property.

5. $f(x) = g[h(x)]$

$\therefore f(-x) = g[h(-x)] = g[h(x)]$

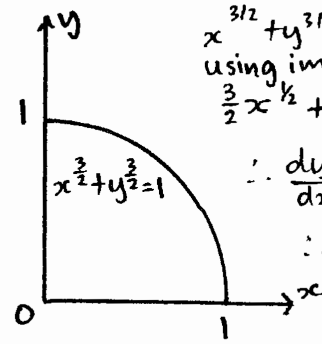
(since $h(x)$ is even $\therefore h(-x) = h(x)$)

$\therefore f(-x) = f(x)$

$\therefore f(x)$ is an even function.

EXERCISE: 1.9

1.



$x^{3/2} + y^{3/2} = 1$

using implicit differentiation

$\frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}y^{\frac{1}{2}}\frac{dy}{dx} = 0$

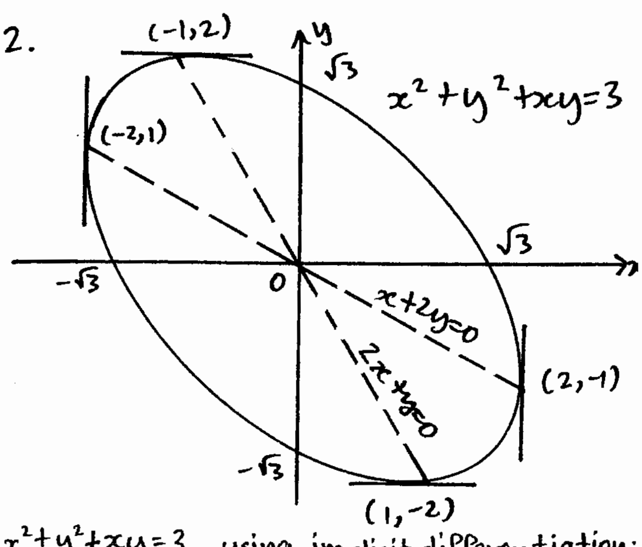
$\therefore \frac{dy}{dx} = \frac{-x^{1/2}}{y^{1/2}}$

$\therefore \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{\frac{1}{2}}$

$\therefore$ The tangent at the critical point (0,1) is horizontal.

$\therefore$ The tangent at the critical point (1,0) is vertical.

2.



$x^2+y^2+xy=3$ using implicit differentiation:

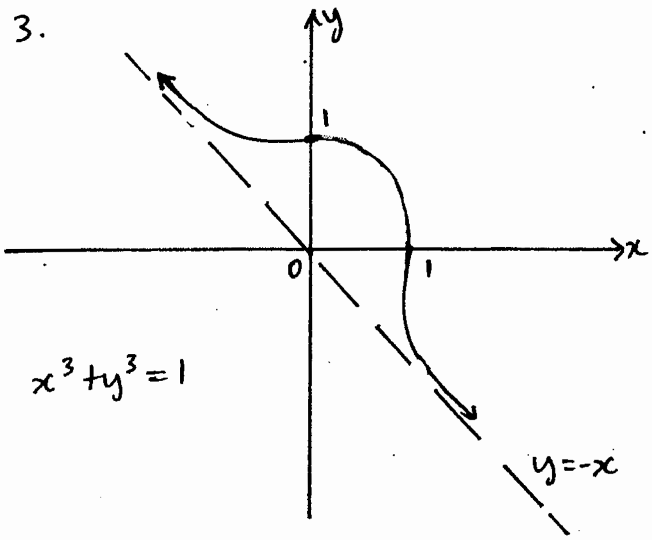
$2x + 2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$(2y+x)\frac{dy}{dx} = -(2x+y)$ $\therefore \frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$

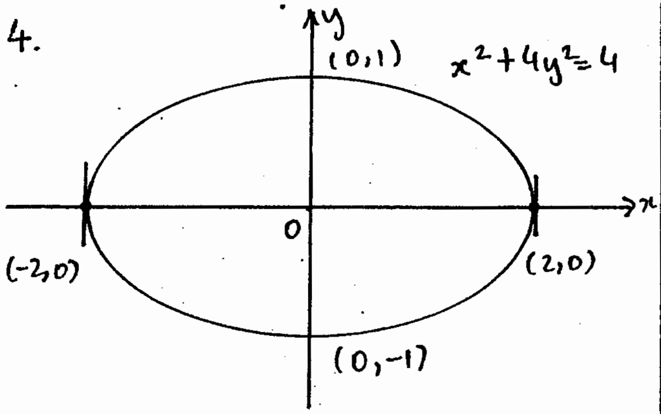
$\therefore$ The tangent at the critical points (-1,2) and (1,-2) are horizontal.

$\therefore$ The tangent at the critical points (-2,1) and (2,-1) are vertical.

3.

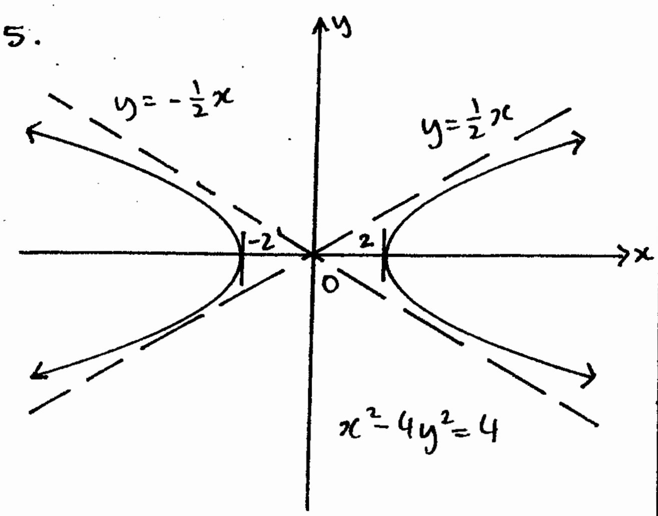


4.



$\therefore$ The vertical tangents $(-2,0)$ and $(2,0)$ are critical points.

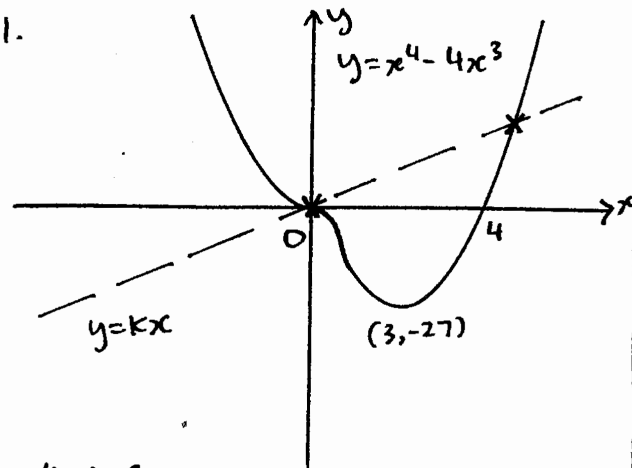
5.



$\therefore$ The vertical tangents $(2,0)$ and $(-2,0)$ are critical points.

EXERCISE: 1·10

1.

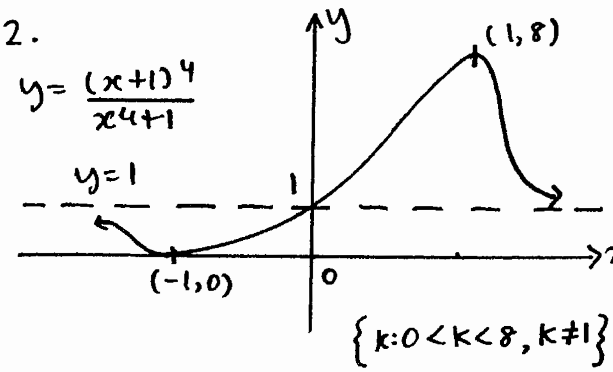


$x^4-4x^3=kx$

The roots of this equation are the points of intersection of $y=kx$ and $y=x^4-4x^3$.

$\therefore$ From the graph we can see that there are 2 real roots for the equation, one of them $(0,0)$ and the other has co-ordinates $x>4$, $y>0$.

2.

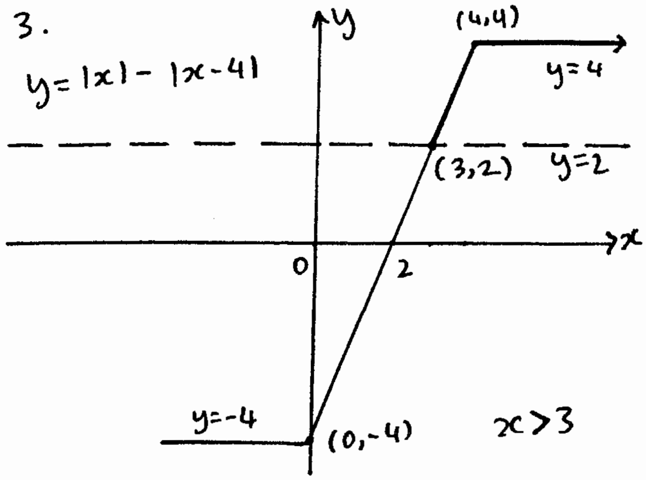


$$\frac{(x+1)^4}{x^4+1}=k$$

The roots of this equation are the points of intersection of $y=k$ and $y=\frac{(x+1)^4}{x^4+1}$.

$\therefore$ From the graph we can see that there are 2 real roots for the equation when $0<k<8$ except $k=1$.

3.



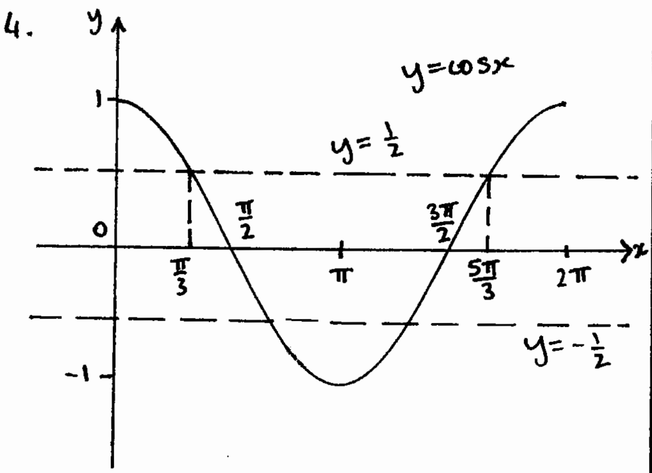
$y=|x|-|x-4|$

$$\therefore y=\begin{cases}-4; & x\le 0\\ 2x-4; & 0<x<4\\ 4; & x\geqslant 4\end{cases}$$

$|x|-|x-4|>2$

From the graph the solution is the shaded part i.e. $x>3$

4.



(a) $\cos x\le\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\cos x=\cos\frac{\pi}{3}$

$\therefore x=\frac{\pi}{3}+2k\pi$ or $x=-\frac{\pi}{3}+2k\pi$

for $k=0$, $x=\frac{\pi}{3}$ $\quad$ for $k=1$, $x=\frac{5\pi}{3}$

$\therefore$ The solutions are $x=\frac{\pi}{3},\frac{5\pi}{3}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph the solution is $\frac{\pi}{3}\le x\le\frac{5\pi}{3}$.

(b) $|\cos x|\le\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \cos x=\frac{1}{2}$ or $\cos x=-\frac{1}{2}$

for $\cos x=\frac{1}{2}$, $\therefore \cos x=\cos\frac{\pi}{3}$

$\therefore x=\frac{\pi}{3}+2k\pi$ or $x=-\frac{\pi}{3}+2k\pi$

for $k=0$, $x=\frac{\pi}{3}$ $\quad$ for $k=1$, $x=\frac{5\pi}{3}$

for $\cos x=-\frac{1}{2}$, $\therefore \cos x=\cos\frac{2\pi}{3}$

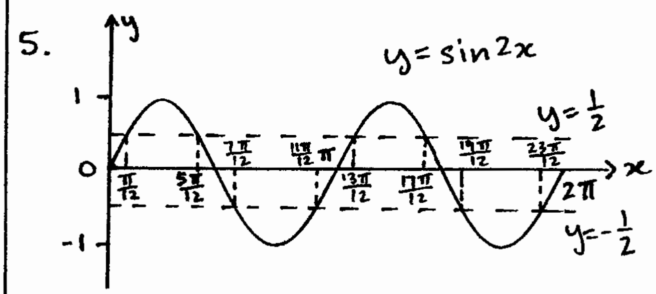
$\therefore x=\frac{2\pi}{3}+2k\pi$ or $x=-\frac{2\pi}{3}+2k\pi$

for $k=0$, $x=\frac{2\pi}{3}$ $\quad$ for $k=1$, $x=\frac{4\pi}{3}$

$\therefore$ The solutions are $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph, the solutions are $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ and $\frac{4\pi}{3}\le x\le\frac{5\pi}{3}$.

5.



(a) $\sin 2x\geqslant\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \sin 2x=\sin\frac{\pi}{6}$

$\therefore 2x=\frac{\pi}{6}+2k\pi$ or $2x=\frac{5\pi}{6}+2k\pi$

$\therefore x=\frac{\pi}{12}+k\pi$ $\quad$ $x=\frac{5\pi}{12}+k\pi$

for $k=0$, $x=\frac{\pi}{12}$ $\quad$ for $k=0$, $x=\frac{5\pi}{12}$

$k=1$, $x=\frac{13\pi}{12}$ $\quad$ $k=1$, $x=\frac{17\pi}{12}$

$\therefore$ The solutions are $x=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph, the solutions are $\frac{\pi}{12}\le x\le\frac{5\pi}{12}$ and $\frac{13\pi}{12}\le x\le\frac{17\pi}{12}$.

(b) $|\sin 2x|\geqslant\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \sin 2x=\frac{1}{2}$ or $\sin 2x=-\frac{1}{2}$

for $\sin 2x=\frac{1}{2}$, $\therefore \sin 2x=\sin\frac{\pi}{6}$

$\therefore 2x=\frac{\pi}{6}+2k\pi$ or $2x=\frac{5\pi}{6}+2k\pi$

$\therefore x=\frac{\pi}{12}+k\pi$ $\quad$ $x=\frac{5\pi}{12}+k\pi$

for $k=0$, $x=\frac{\pi}{12}$ $\quad$ for $k=0$, $x=\frac{5\pi}{12}$

for $\sin 2x=-\frac{1}{2}$, $\therefore \sin 2x=\sin(-\frac{\pi}{6})$

$\therefore 2x=-\frac{\pi}{6}+2k\pi$ or $2x=\frac{7\pi}{6}+2k\pi$

$\therefore x = -\frac{\pi}{12} + k\pi$ or $x = \frac{7\pi}{12} + k\pi$

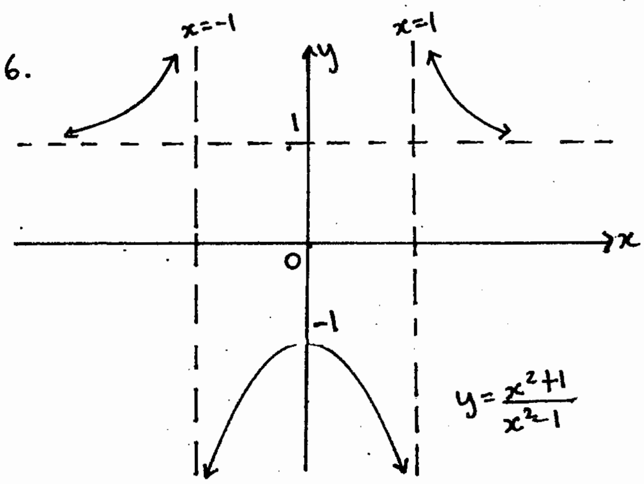
for $k=1$, $x = \frac{11\pi}{12}$ for $k=0$, $x = \frac{7\pi}{12}$

$k=2$, $x = \frac{23\pi}{12}$ $k=1$, $x = \frac{19\pi}{12}$

$\therefore$ The solutions are $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$.

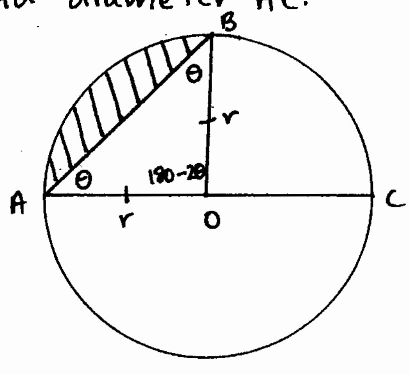
$\therefore$ From the graph the solutions are $\frac{\pi}{12} \le x \le \frac{5\pi}{12}$, $\frac{13\pi}{12} \le x \le \frac{17\pi}{12}$, $\frac{7\pi}{12} \le x \le \frac{11\pi}{12}$, $\frac{19\pi}{12} \le x \le \frac{23\pi}{12}$.

6.



From the graph, the solution is $-1 < x < 1$.

7. Considering a circle O of radius r and diameter AC.



$AO = OB$ (equal radii of circle)

$\therefore \Delta ABO$ is isosceles.

$\angle BAO = \theta$ (data)

$\therefore \angle ABO = \theta$ (base angles of isosceles ΔABO are equal.)

$\therefore \angle AOB = 180 - 2\theta$ (angle sum ΔABO)

A(minor segment) $= A$(sector) $- A(\Delta ABO)$

$= \frac{1}{2} r^2(\pi - 2\theta) - \frac{1}{2} r^2 \sin(\pi - 2\theta)$

$= \frac{1}{2}\pi r^2 - r^2\theta - \frac{1}{2} r^2 \sin 2\theta$

(Note: $\sin(\pi - 2\theta) = \sin 2\theta$)

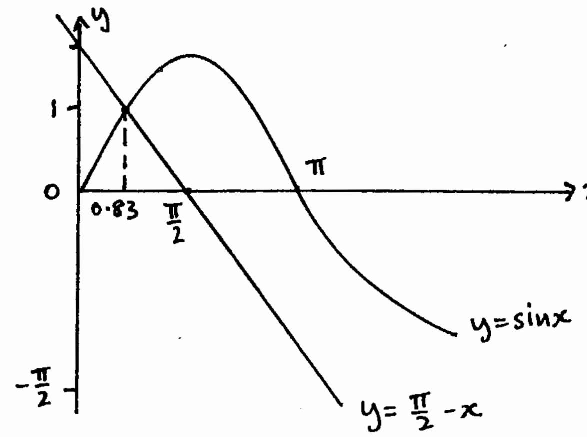
$\therefore \frac{1}{2}\pi r^2 - r^2\theta - \frac{1}{2} r^2 \sin 2\theta = \frac{1}{4}\pi r^2$

(dividing by r^2)

$\therefore \frac{\pi}{2} - \theta - \frac{1}{2}\sin 2\theta = \frac{\pi}{4}$

$\frac{\pi}{4} - \theta - \frac{1}{2}\sin 2\theta = 0$ $\therefore \sin 2\theta = \frac{\pi}{2} - 2\theta$

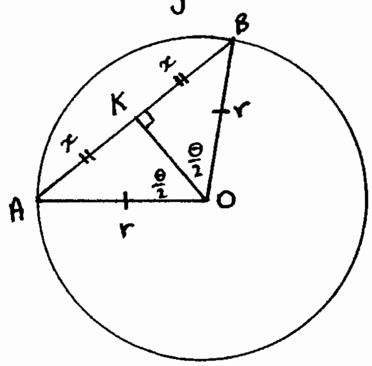
Let $2\theta = x$ $\therefore \sin x = \frac{\pi}{2} - x$



From the graph, $x = 0.83$

$\therefore 2\theta \doteqdot 0.83$ $\therefore \theta \doteqdot 0.4$

8. Considering a circle O of radius r.



Construct KO where KO bisects $\angle AOB$ but since $\angle AOB = \theta$ (data)

$\therefore \angle KOA = \frac{\theta}{2} = \angle KOB$ let $KB = x$ $\therefore KA = x$

P(minor segment) = length arc AB + length chord AB

in ΔKOB, $\sin\left(\frac{\theta}{2}\right) = \frac{x}{r}$ $\therefore x = r\sin\frac{\theta}{2}$

$\therefore AB = 2r\sin\left(\frac{\theta}{2}\right)$

$\therefore P$(minor segment) $= r\theta + 2r\sin\left(\frac{\theta}{2}\right)$

$\therefore r\theta + 2r\sin\frac{\theta}{2} = \pi r$

(dividing by r)

$\therefore \theta + 2\sin\frac{\theta}{2} = \pi \quad \therefore 2\sin(\frac{\theta}{2}) = \pi - \theta$

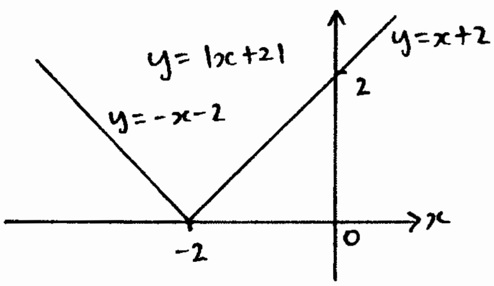
Let $\frac{\theta}{2} = x \quad \therefore \sin x = \frac{\pi}{2} - 2x$

From the graph $x = 0.83$

$\therefore \quad \frac{\theta}{2} \doteqdot 0.83 \quad \therefore \theta \doteqdot 1.7$

DIAGNOSTIC TEST 1

1.(a)

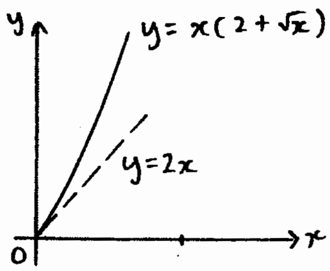


when $x \to -2^-$, $\frac{dy}{dx} = -1$

when $x \to -2^+$, $\frac{dy}{dx} = 1$

$\therefore$ The point $(-2, 0)$ is a critical point where $\frac{dy}{dx}$ is not defined.

(b)



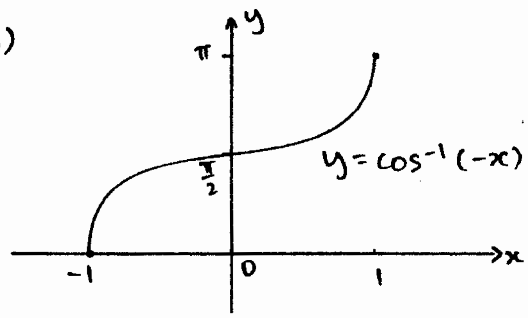
$y = 2x + x\sqrt{x} \quad \therefore y = 2x + x^{3/2}$

$\therefore \frac{dy}{dx} = 2 + \frac{3}{2}x^{1/2}$

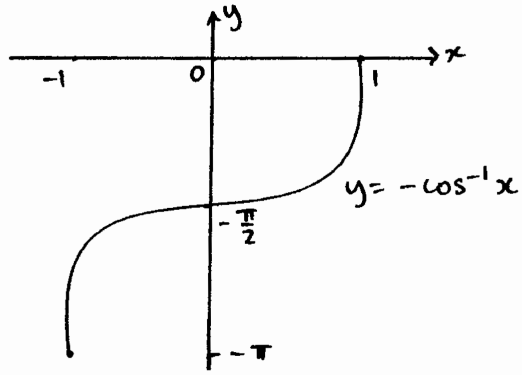
when $x \to 0^+$, $\frac{dy}{dx} = 2$

$\therefore (0,0)$ is a critical point where the tangent at this point is $y = 2x$.

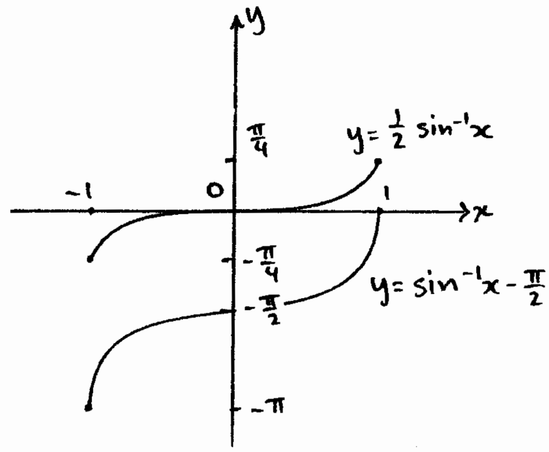
2.(a)



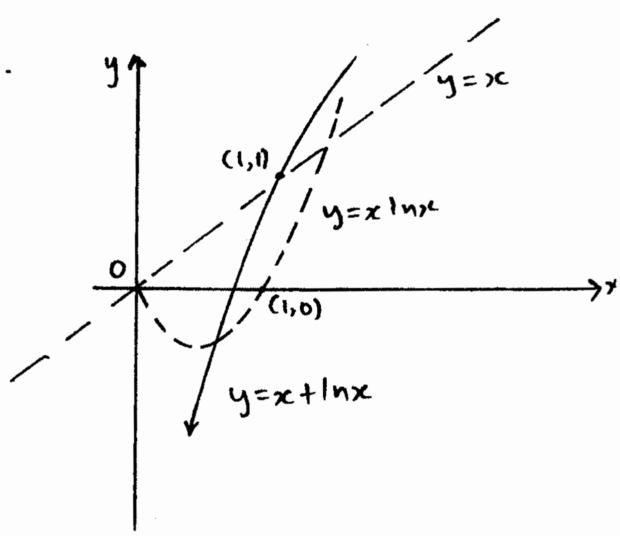
(b)



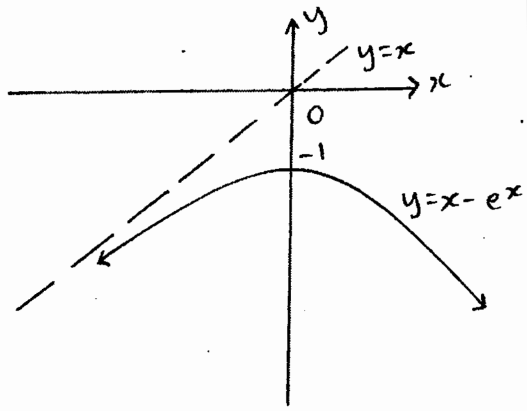
3.



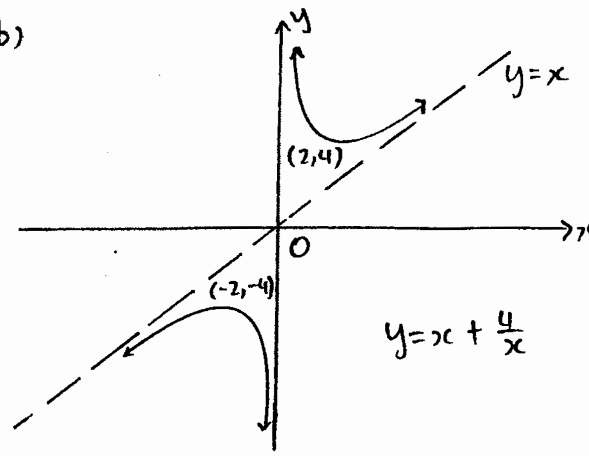
4.



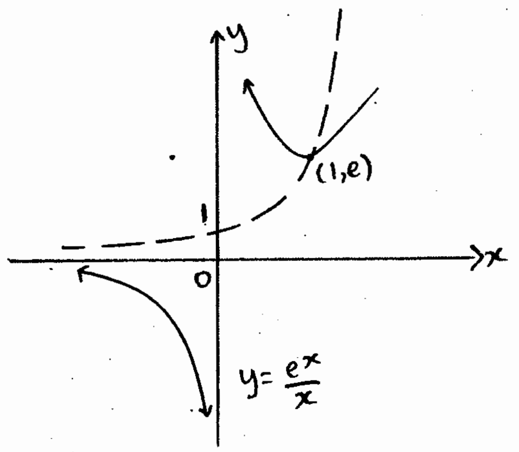
5. (a)



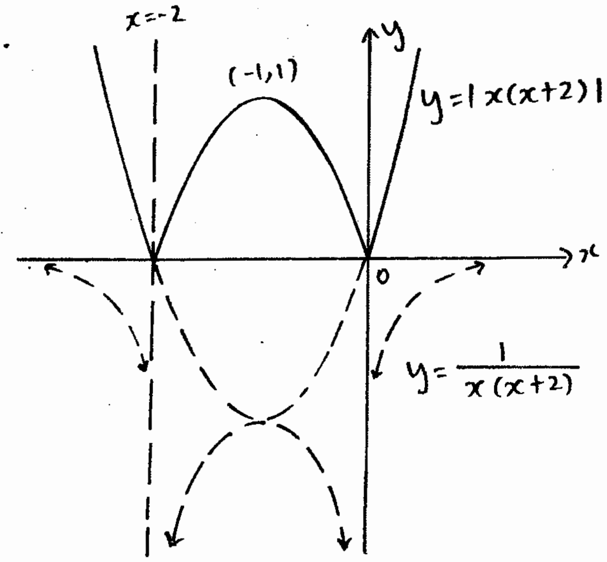
(b)



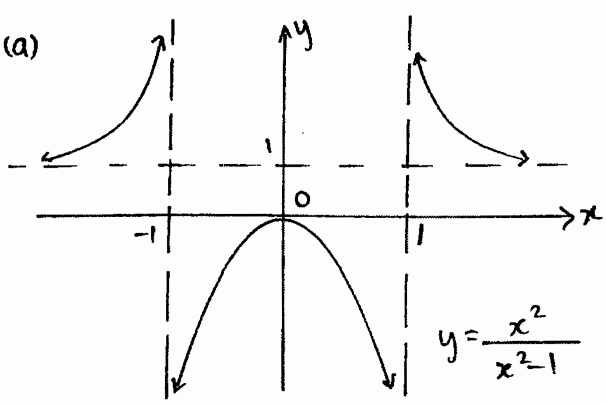
(b)



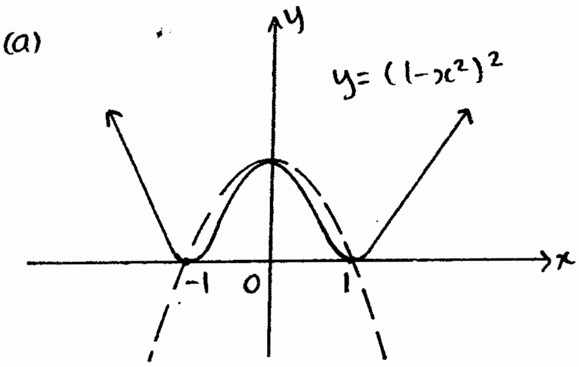
6.



7. (a)

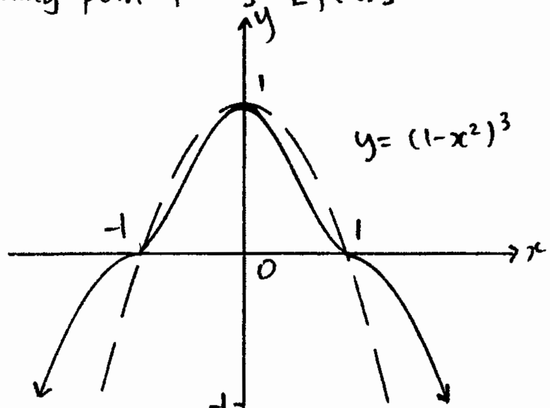


8. (a)



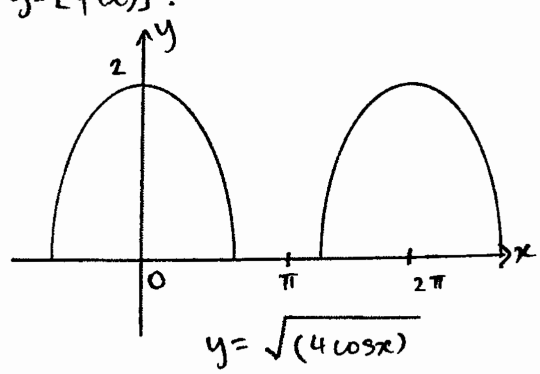
When squaring a function $f(x)$, every intersection with x-axis becomes a minimum turning point for $y= [f(x)]^2$

(b)

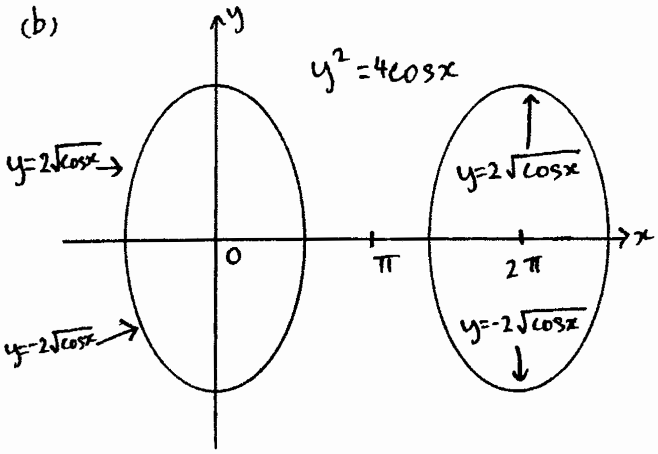


When cubing $y=f(x)$, every intersection with y-axis becomes a horizontal point of inflexion for $y=[f(x)]^3$.

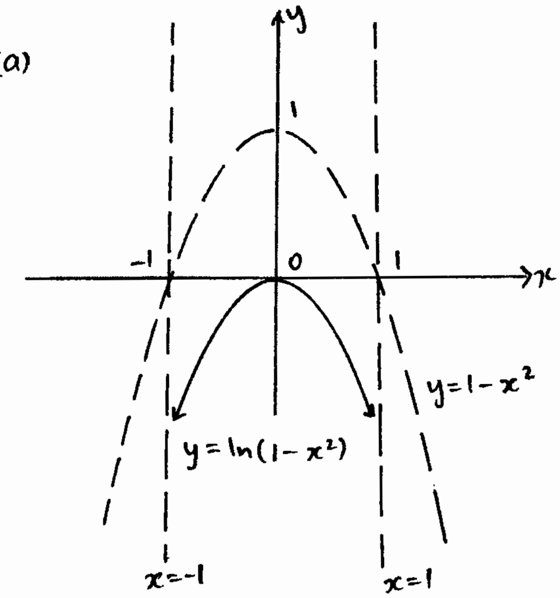
9. (a)



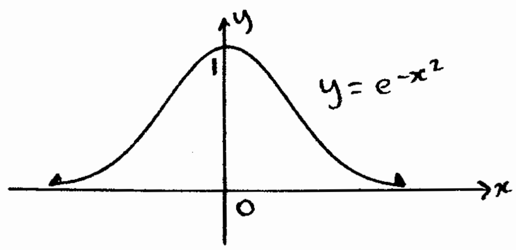
(b)



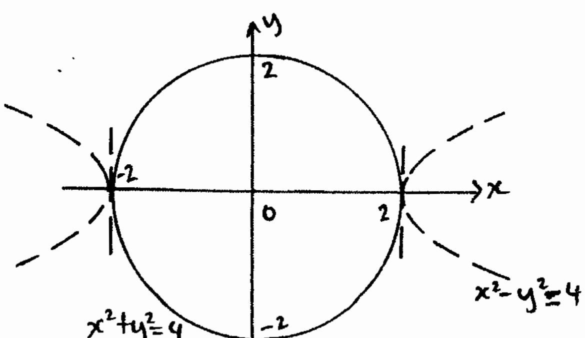
10. (a)



(b)

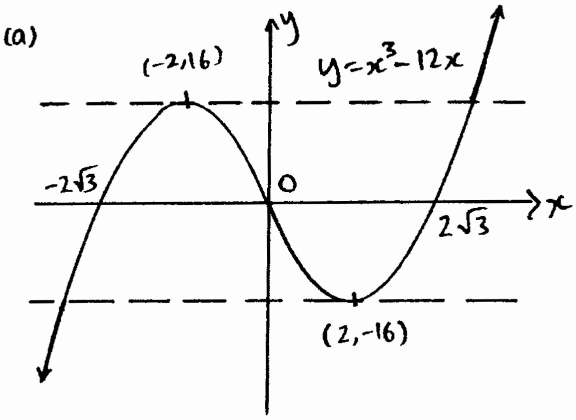


11.



$x^2+y^2=4$ is the equation of a circle and $x^2-y^2=4$ is the equation of a hyperbola.

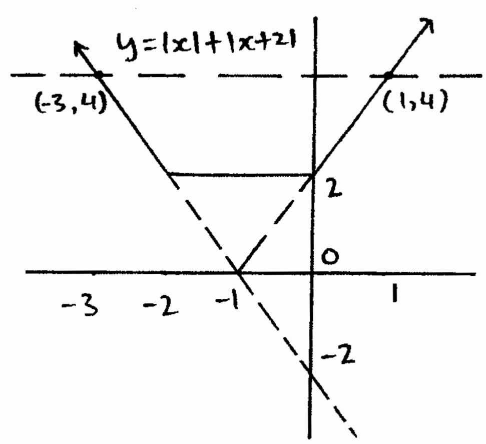
12. (a)



$x^3-12x+k=0$ $\quad\therefore x^3-12x=-k$

$\therefore$ The equation will have one root when $y=-k$ intersects the curve $y=x^3-12x$ only once. From the graph we can see that for $k>16$ or $k<-16$, this equation has only root.

(b) $y=\begin{cases}-2x-2 & \text{for } x<-2\\ 2 & \text{for } -2\le x\le 0\\ 2x+2 & \text{for } x>0\end{cases}$



$|x|+|x+2|=4$ $\quad$ for $x<-2$

$\therefore -2x-2=4$ $\quad\therefore x=-3$

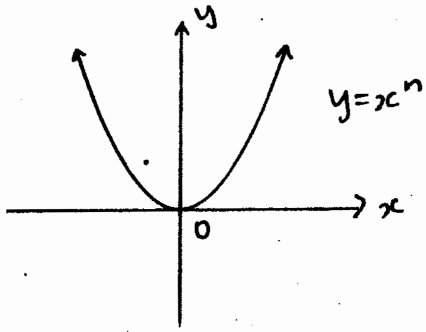
$\therefore (-3,4)$ is a solution.

for $-2\le x\le 0$ $\quad\therefore 2=4$ invalid

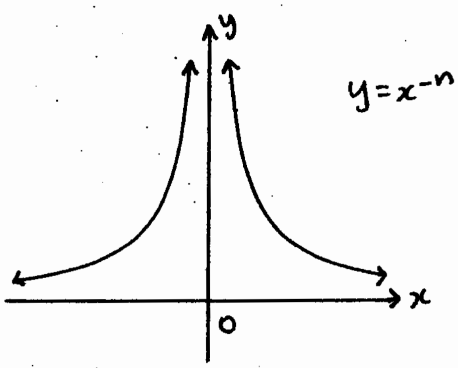
for $x>0$ $\therefore 2x+2=4$ $\therefore x=1$
$\therefore (1,4)$ is the other solution.
from the graph we can see that
$|x|+|x+2|>4$ when $x<-3$ or
$x>1$.

Further Questions 1

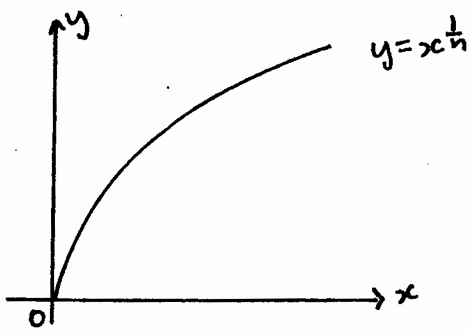
1.(a)



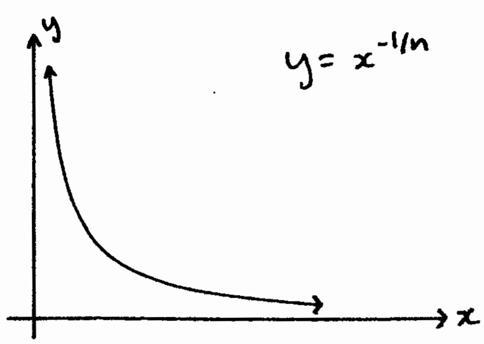
(b)



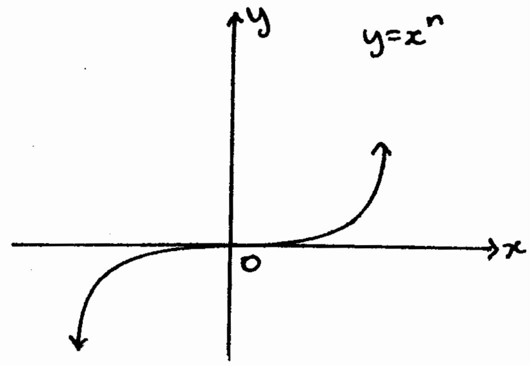
(c)



(d)



2.(a)



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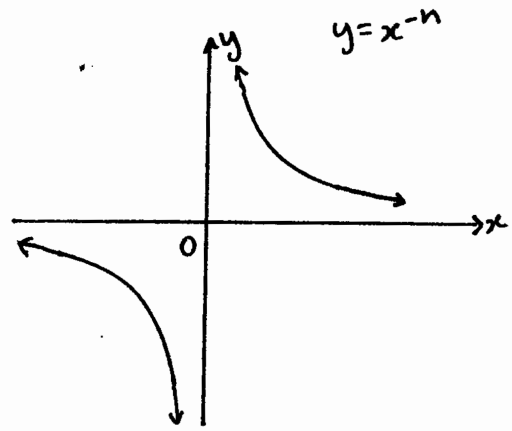

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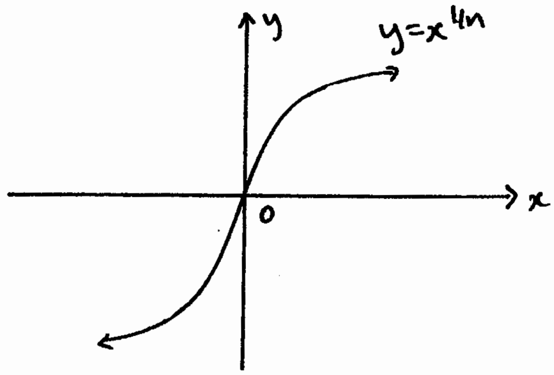

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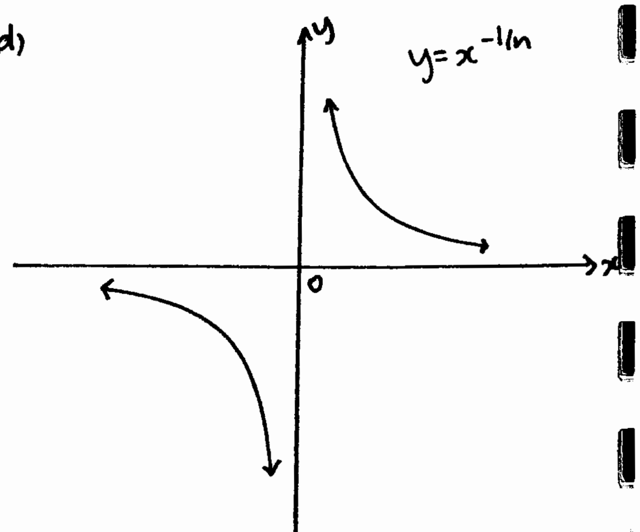

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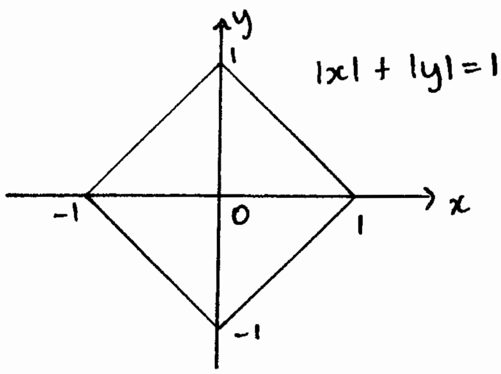

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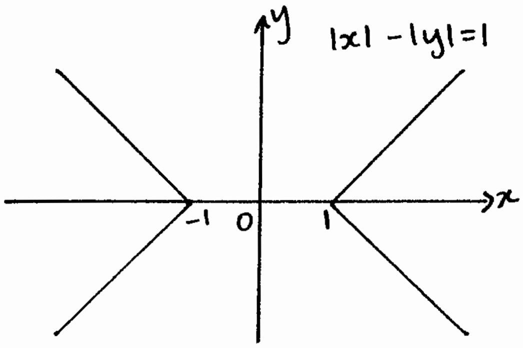

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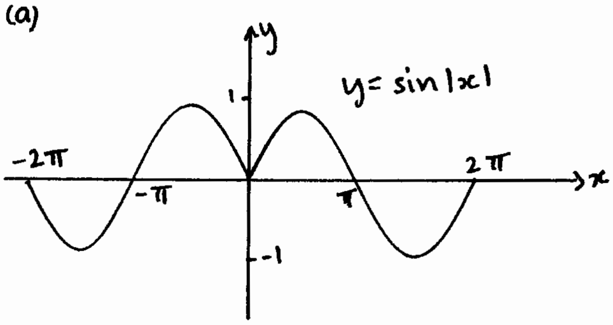

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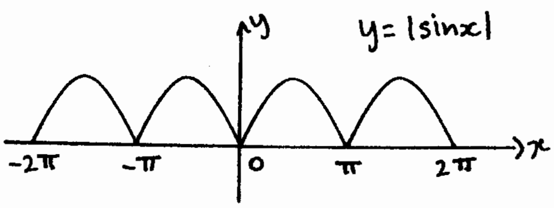
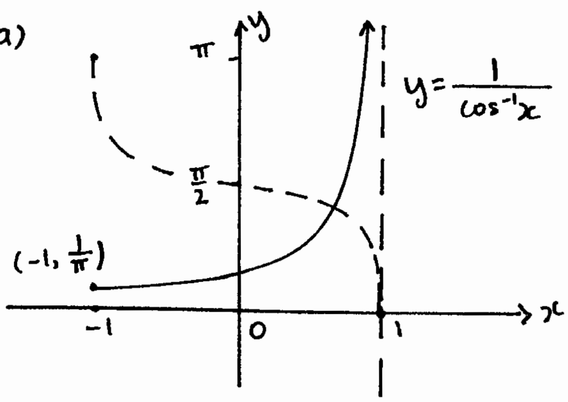


5.(a)



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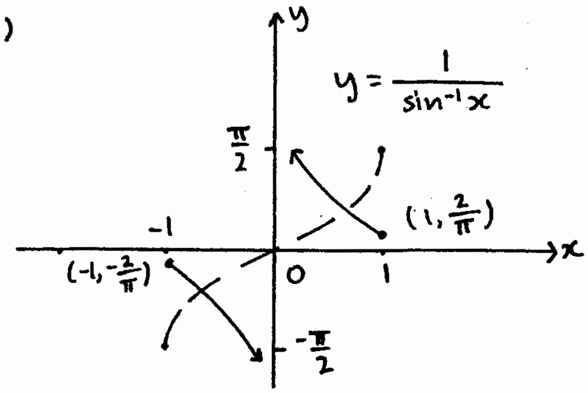

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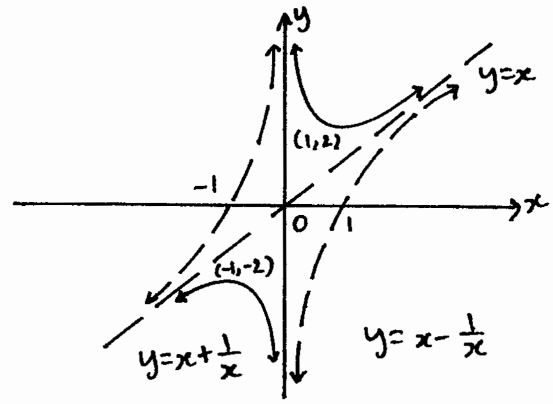

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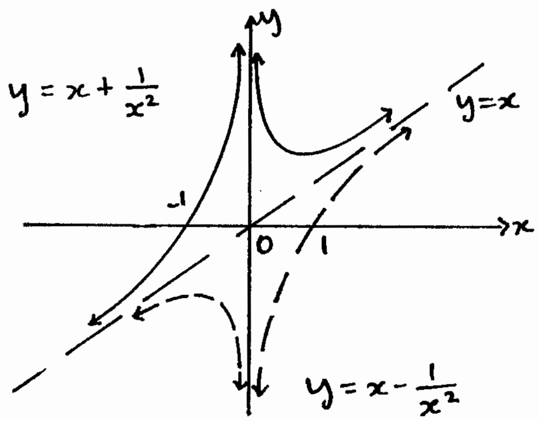

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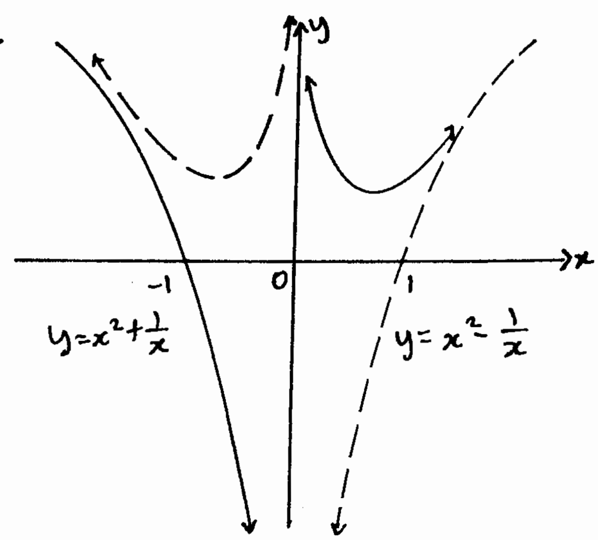

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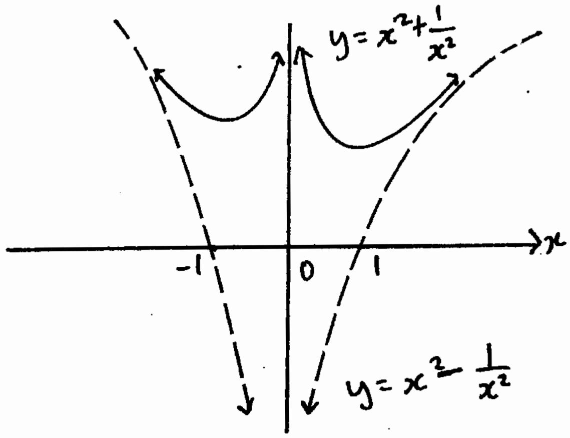

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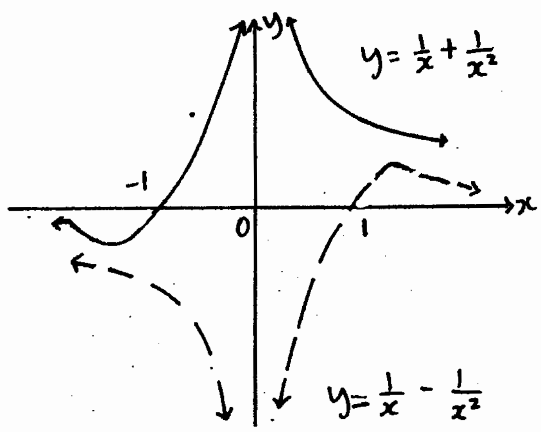
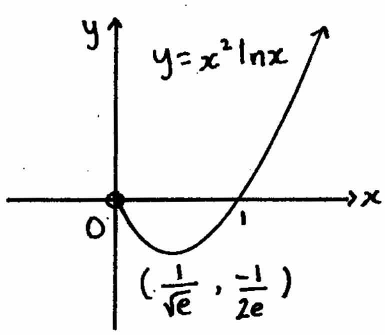


11. (a)



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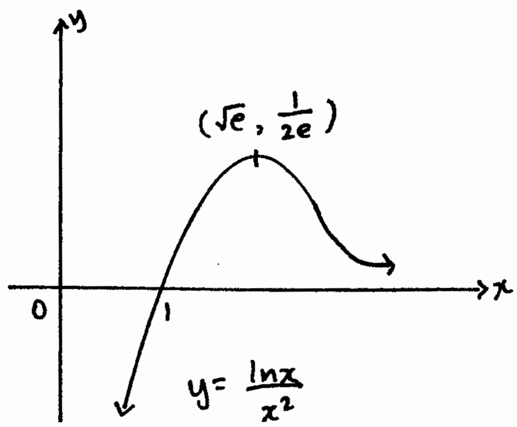
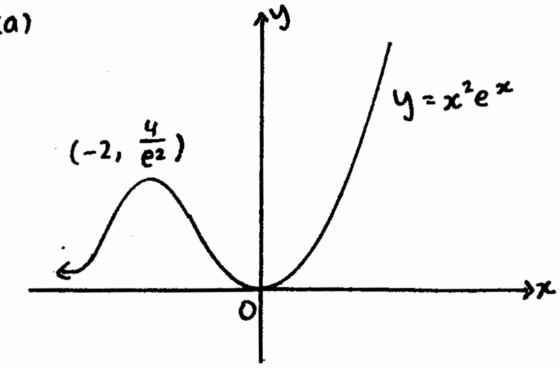


12. (a)



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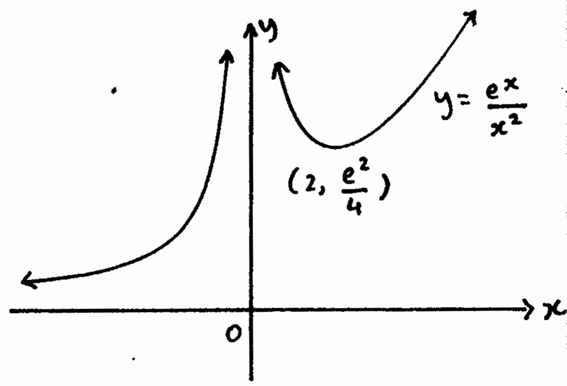
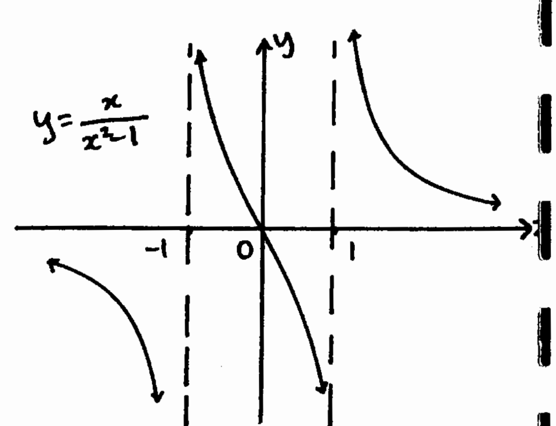


13. (a)



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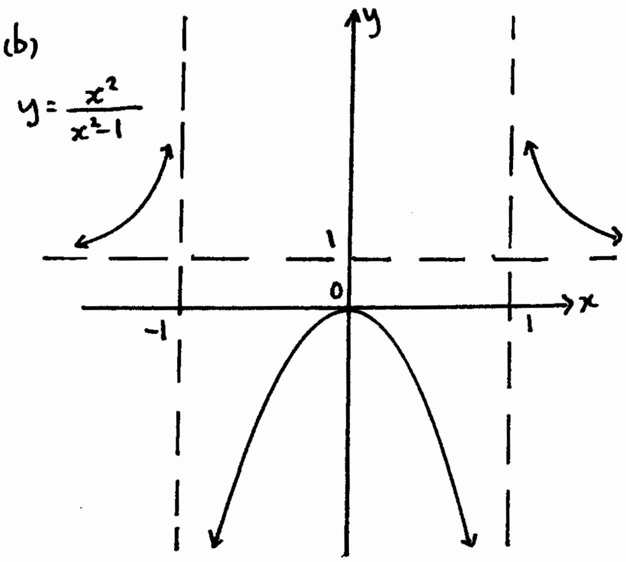
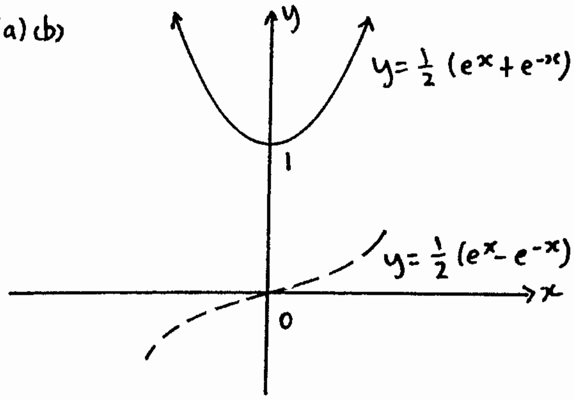
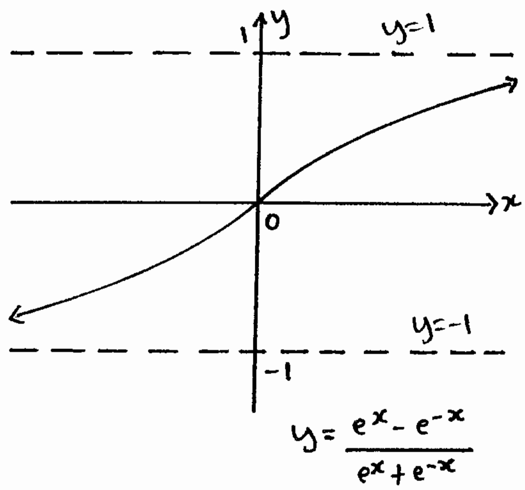


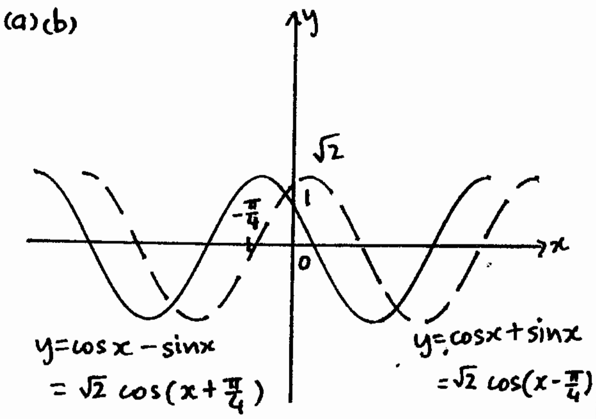
14. (a)(b)



(c)



15. (a)(b)



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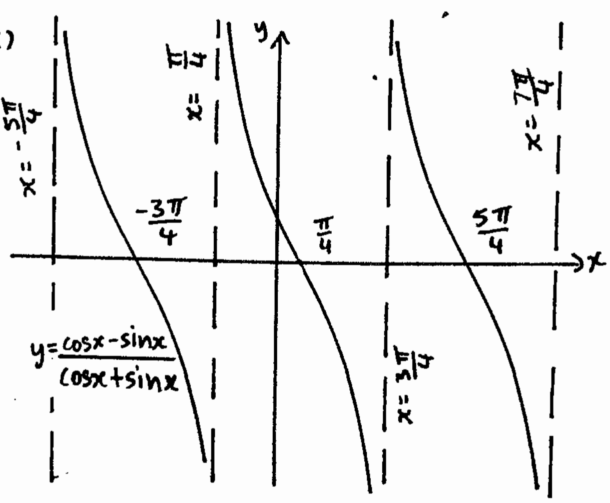

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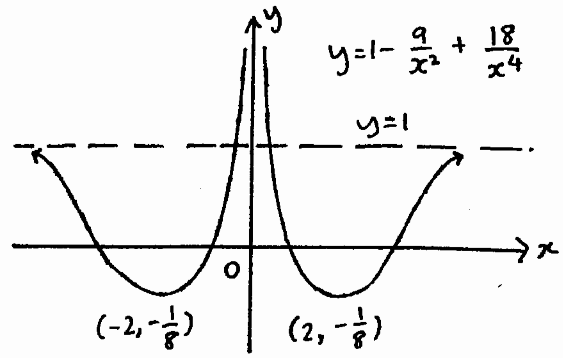
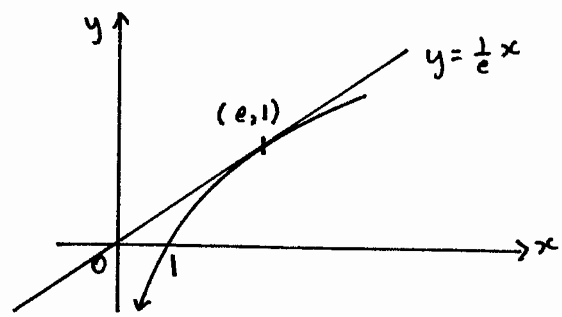


The equation $f(x)=k$ will have 4 roots when the horizontal line $y=k$ crosses the curve 4 times. $\therefore -\frac{1}{8}<k<1$

17.



Equation of a line passing by (0,0) is $y=kx$. Let the line $y=kx$ meet the curve $y=\ln x$ at (x_0, y_0)

$\therefore \ln x_0 = kx_0$ (1)

The gradient of the tangent at (x_0, y_0) on the curve $y=\ln x$ is $\frac{dy}{dx}=\frac{1}{x}$ (grad at x_0 $\therefore m_{tan}=\frac{1}{x_0}$

and since the line $y=kx$ is also tangent at (x_0, y_0) $\therefore k=\frac{1}{x_0}$ (2)

from (1) and (2):

$\ln x_0 = \frac{1}{x_0}\times x_0$ $\therefore \ln x_0 = 1$

$\therefore x_0 = e$ $\therefore y_0 = \ln e = 1$ $\therefore m_{tan}=\frac{1}{e}$ from (0,0)

The equation has two real distinct roots when $0<k<\frac{1}{e}$.

18. $xy(x+y)+16=0$ (1)

$x^2y+xy^2+16=0$

Using implicit differentiation,

$2xy+x^2\frac{dy}{dx}+y^2+2xy\frac{dy}{dx}=0$

$\frac{dy}{dx}(x^2+2xy) = -2xy - y^2$

$\therefore \frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$ (gradient function)

$\frac{dy}{dx} = -1 \quad \therefore \frac{-2xy-y^2}{x^2+2xy} = -1$

$\therefore 2xy + y^2 = x^2 + 2xy$

$\therefore y = \pm x$ sub $y=x$ into (1)

$\therefore 2x^3 + 16 = 0 \quad \therefore x^3 = -8 \quad \therefore x = -2$

$\therefore y=-2$ $\therefore$ The point is $(-2,-2)$.

$\therefore \frac{y+2}{x+2} = -1 \quad \therefore y+2 = -x-2$

$\therefore x+y+4=0$

Now, for $y=-x$

$-2x^3+16=0 \quad \therefore x^3=8 \quad \therefore x=2 \quad \therefore y=-2$

$\therefore$ The point is $(2,-2)$

$\therefore \frac{y+2}{x-2} = -1 \quad \therefore y+2 = -x+2$

$\therefore x+y=0$

By substituting into (1) we get:

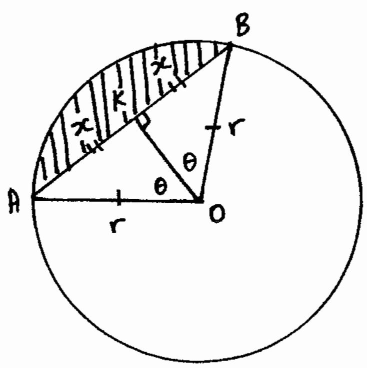
$16=0$ invalid

$\therefore$ There is no double root.

For $x+y=0$

$\therefore$ The only tangent to the curve is $x+y+4=0$.

19.



Perimeter of minor segment= Arc AB + chord AB

using ΔKOB, $\sin\theta = \frac{x}{r}$ $\therefore x = r\sin\theta$

$\therefore AB = 2r\sin\theta$

$\therefore$ Perimeter of minor segment $= r\theta + 2r\sin\theta$

$\therefore$ Perimeter DOAB is $2r + 2r\sin\theta$.

$\therefore 2r\theta + 2r\sin\theta = K(2r + 2r\sin\theta)$

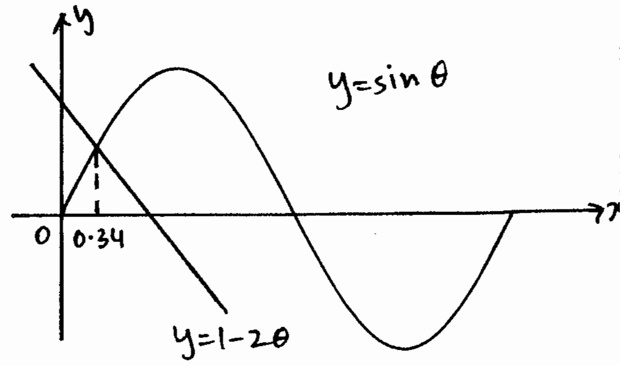
Dividing by $2r$

$\therefore \theta + \sin\theta = k + k\sin\theta$

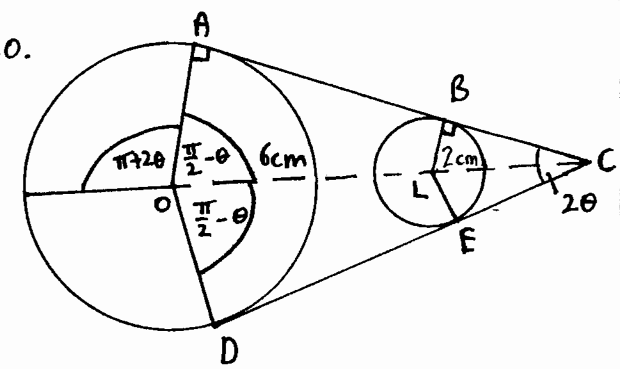
$k\sin\theta - \sin\theta + k = \theta \quad \therefore (k-1)\sin\theta + k = \theta$

for $k=\frac{1}{2}$ $\therefore -\frac{1}{2}\sin\theta + \frac{1}{2} = \theta$

$\therefore \sin\theta - 1 = -2\theta \quad \therefore \sin\theta = 1-2\theta$



20.



Using ΔAOC, $\tan(\frac{\pi}{2}-\theta) = \frac{AC}{6}$ $\therefore AC = 6\cot\theta$

Using ΔBLC, $\tan(\frac{\pi}{2}-\theta) = \frac{BC}{2}$ $\therefore BC = 2\cot\theta$

$\therefore AC - BC = 4\cot\theta$ (length of tangent AB)

$\therefore$ Length of both tangents is $8\cot\theta$.

Arc $AD = 6(\pi+2\theta)$; Arc $BE = 2(\pi-2\theta)$

$\therefore$ Total length is $8\cot\theta + 6\pi + 12\theta + 2\pi - 4\theta = 44$cm

$\therefore 8\cot\theta + 8\pi + 8\theta = 44$cm $\therefore \cot\theta + \pi + \theta = 5.5$